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AN INTEGRATED APPROACH FOR THE MERGER OF SMALL AND MEDIUM-SIZED INDUSTRIAL UNITS

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Abstract. Considering the importance of small and medium-sized industrial units for economic growth, social cohesion, regional and local development, this study presents a model for the merger so that they can make use of each other's capacities and facilities to achieve higher efficiency levels. The involved criteria have been chosen using the SCOR model with the consideration of sustainability, resilience and agility criteria in each part of the supply chain network. PCA has been used to reduce the dimensionality and the efficiency of units has been determined by network DEA. Next, a mathematical model has been used to determine the best combination for merger. The model chosen for the finalization of the merger process is inverse network DEA, which tries to determine the final inputs of the merged units for a specific target. In addition to theoretical benefits, the results have practical applications. The results can give supply chain partners a common language for better communication and help them settle on standardized definitions. The model has been implemented using real-world data gathered from other articles, which pertain to 26 stone industries of Iran. The DEA model and the mathematical model have been solved through GAMS and the PCA approach through MATLAB.

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1. Introduction

All governments around the world acknowledge the importance of small and medium-sized enterprises (SMEs) and their contribution to economic growth, social cohesion, job creation, and regional and local development. Recognizing these potentials, many developing countries have put the growth of SMEs at the top of their agenda, offering them special support, protections, and other incentives. Considering the importance of industrial SMEs, this study concerns the subject of the merger of industrial SMEs for the purpose of nurturing closer cooperation, sharing competitive advantages, sharing experiences and facilities, using joint capital for modernizing equipment and practices, developing more resistance to unanticipated changes, bringing product specifications closer to international standards, and improving the quality of products. Other advantages of such mergers include the integration and joint use of equipment and machinery, the integration of sales units, which eliminates unnecessary competition between them, the use of joint capital to import equipment, etc.

This study makes use of the Supply Chain Operations Reference (SCOR) model, which is one of the most reliable tools for supply chain evaluation, enabling managers to assess complex processes in their supply chains.

Keywords. Supply chain, exports, inverse data envelopment analysis (IDEA), PCA, SCOR, Sustainability.

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It can also serve as a diagnostic tool for supply chain management, helping users to understand the processes involved in a business organization and identify the critical features that lead to customer satisfaction [33]. In this study, the SCOR model is used for the performance evaluation of units. After identifying the criteria for measuring performance, resilience, and agility, Principal Component Analysis (PCA) is used to reduce the dimensionality of the problem, and a version of Network Data Envelopment Analysis (NDEA) is used to determine the efficiency of units. While most articles in the literature concerning this subject have used the traditional Data Envelopment Analysis (DEA) models, in order to succeed in international markets, even the smallest sub-processes of industrial units must be taken into account during performance evaluations. This can be done by the use of inverse NDEA to analyze the changes in not only inputs and outputs but also intermediates. In the next stage of the study, the developed mathematical model is used to determine the best combination of units for merger subject to constraints such as budget, distance, and the number of employees, and with objectives such as sales maximization. For the finalization of the merger process, the inverse NDEA is utilized to determine the final inputs and outputs of the merged units for a specific performance target. Contrary to traditional DEA, which attempts to measure the efficiency of units in terms of an objective function, inverse DEA tries to determine the best possible output (input) for a given input (output) for a constant objective function value. In other words, it determines how much input should be consumed to maintain or improve efficiency at a certain output level.

A variety of different strategies can be adopted for improving profitability through mergers, which depend on whether the merger is vertical or horizontal. The present study is mainly focused on horizontal mergers. In this type of merger, one company merges with or takes control of another company that offers similar products or operates in the same industry and at the same stage of production. In this type of merger, merging companies tend to be direct competitors. This type of merger has several benefits, the most important of which is the elimination of competition, which helps increase market share, revenue, and profits. Furthermore, the increased firm size tends to result in a reduction in average costs because of higher production volume and economies of scale. This type of merger also leads to higher cost-effectiveness by reducing redundant activities and operational processes (*e.g.* in management, advertising, procurement, and marketing domains). Horizontal integrations tend to have an impact not only on consumer welfare through price changes, but also on consumer behavior through changes in non-price characteristics such as product quality.

In addition to their theoretical benefits, the results of this study also have some practical applications. Most importantly, the results can help various industries including the stone industry to communicate better with supply chain partners and settle on standardized definitions for processes, process elements, and criteria. From the perspective of the research domain, the present study explores not only the subject of the merger of industrial units but also the selection of the optimal combination of units for the merger. In fact, in this study, the units that are to be merged are determined with the consideration of all applying limitations and conditions as well as the purpose of the merger.

The second section of this article provides a review of the literature on the subject. The third section examines the theoretical foundations and the fourth section presents the proposed model. The fifth section presents the conclusions and provides a number of suggestions for future research.

2. Literature review

2.1. Data envelopment analysis and principal component analysis

Adler and Golany have used a combination of data envelopment analysis and principal component analysis to reduce dimensions when the number of inputs and outputs is much larger than the decision units [1]. Azadeh examined an integrated framework for evaluating and ranking manufacturing systems – based on management and organizational performance indicators – for Iran's telecommunications sector [9]. The first major component has the highest variance in the sample data. The second component has the second level of variance and so the principal components are calculated [3].

Principal components can be used to replace all inputs or outputs simultaneously or to replace specific groups of variables [2]. Tavakoli and Shirouyehzad used PCA-DEA to evaluate the performance of the steel company based on its human capital management [35]. Fu *et al.*, in their research, concluded that by combining PCA and DEA in evaluating the performance of energy projects, the results improved in comparison to those who used only the simple DEA method [20].

PCA is a multivariate technique used to analyze the relationships of variables and explains the variables based on their components [24]. To avoid subjectivity in selecting the index, PCA (principal component analysis) was used to reduce the dimensions and obtained comprehensive and objective indicators, followed by performance evaluation and analysis [34]. Jothimani *et al.* used financial ratios as input and output performance evaluation parameters by PCA-DEA in an Indian stock market study [27]. Jakaitienė *et al.* proposed the PCA-DEA evaluation method to evaluate the performance of the European education system [26]. In a paper developed in 2020, a combination of data envelopment analysis and principal component analysis was used to evaluate the performance of 100 companies based on their 2015 financial statements [11].

In 2020, an article was developed with the aim of considering carbon emission factors in the evaluation of logistics performance. In this paper, two approaches of principal component analysis and DEA cover approach were used [17]. In another paper, the financial performance of 46 financial institutions was calculated using a combination of two approaches: data envelopment analysis and principal component analysis [13]. In 2021, the confidence area method was used in data envelopment analysis and combined with the PCA approach to evaluate the energy security performance of 125 countries for 21 consecutive years [37]. In addition, in 2022, with the aim of evaluating the efficiency of stone industrial units, a combination of two methods, DEA and PCA has been used [31]. In another research, in order to analyze export conditions in the stone industry, in one of the stages, a combination of PCA and DEA approaches was used [32].

2.2. Inverse data envelopment analysis

Unlike standard data envelopment analysis, which aims to find performance scores, inverse data envelopment analysis, once performance is known, aims to determine the levels of inputs and outputs needed to achieve the desired performance score. The idea of inverse data envelopment analysis was first introduced by Wei to evaluate inputs and outputs for a resource allocation problem [36].

The source of inverse data envelopment analysis is reverse optimization. Unlike normal optimization, in which the goal is to find the optimal solution, in a reverse optimization, a practical solution is given that is not necessarily desirable [4].

Then, Jahanshahloo *et al.* developed an inverse data envelopment analysis model. The purpose of their proposed model was to estimate DMU0 output with an increase in input. In this study, a sensitivity analysis was proposed in the inverse data envelopment analysis model on inputs and outputs [25]. Alinejad examined how to modify the number of outputs and inputs to keep the relative efficiency score constant and used the multi-objective interactive linear programming method [5]. Using reverse data envelopment analysis, Lin measured the performance and revenue scores of a number of chain stores in Taiwan [29].

Lertworasirikul *et al.* discussed the possibility of simultaneously increasing some outputs and decreasing other outputs using inverse data envelopment analysis [28]. Hosseinzadeh Lotfi *et al.* developed an inverse data envelopment analysis model to estimate output if some inputs increase [30]. In 2017, Ghiyasi also addressed the issue of inverse data envelopment analysis when price data is known. In fact, the proposed models guarantee not only the technical efficiency but also the labor cost (revenue) of all DMUs. In this paper, the theoretical foundations of this problem are developed and a MOLP structure is developed [22]. Chen *et al.* proposed an inverse data envelopment analysis model to deal with adverse outputs [15].

In 2018, Emrouznejad *et al.* examined the solution to the problem of allocating greenhouse gas quotas set by the government in the Chinese manufacturing industry to different regions of China with the inverse data envelopment analysis approach [18]. In 2021, an article developed a new model of inverse data envelopment analysis with undesirable outputs to ensure the achievement of the safety goal at the current technical level and reduce the number of fatalities in road accidents. Two safety objectives and two additional objectives are defined and then their realization paths are determined by using the new inverse DEA model, respectively. The model calculated the optimal adjustments of inputs, desirable and undesirable outputs, and the corresponding realization paths are thus obtained to achieve these objectives [16].

Because of limitation of the change range of outputs/inputs on the assumption of VRS, Chen *et al.* provided the specific optimization scheme for the given efficiency level; then the practicability and effectiveness of inverse DEA method were improved [14].

2.3. Merging of industrial units

Gattoufi *et al.* developed the concept of inverse data envelopment analysis in the concept of unit merger analysis to find the optimal levels of inputs and outputs required from entity mergers. Using inverse data envelopment analysis, the authors enabled the integrated entity to achieve an efficient goal [21].

In a 2016 article, the merging of decision-making units despite negative variables was discussed. In this model, the integrated entity can adjust its inputs and outputs to achieve a specific efficiency goal. The proposed method examines the maximum improvement in negative output of DMUs after merging [7]. Amin *et al.* also developed two-stage inverse data envelopment analysis (DEA) models for estimating potential gains from bank mergers for the top US commercial banks. The results showed additional intermediate and final outputs at different predefined target levels of technical efficiencies. The paper confirms that there are financial gains to improving technical efficiency as the merged bank improves its optimal mix of inputs at higher efficiency levels [8]. Estimation of operating efficiency gains from possible M&A between banks is another topic that has been developed by Al Tamimi and *et al.* Their empirical analysis was based on conventional and Islamic banks located in the MENA region and Turkey [6].

Ghobadi studied the problem of merging units in the presence of interval data by considering inverse data envelopment analysis. An insightful method was provided to obtain the minimum and maximum achievable lower and upper bounds of the efficiency interval of the merged unit [23].

3. Problem definition and modeling

One of the methods widely used in the performance evaluation of supply chains is the Network Data Envelopment Analysis (NDEA). NDEA measures the performance and efficiency of a unit whilst considering all the components involved based on the most detailed criteria. In this study, this method is used to determine the efficiency of industrial units. Given the large number of criteria considered in this study, which greatly undermine the ability of NDEA models to provide acceptable results, Principal Component Analysis (PCA) is used for dimensionality reduction in order to increase the method's differentiation power by significantly reducing the amount of data fed to the model. The CRS version of NDEA is then used to determine the efficiency of sub-networks and ultimately the efficiency of the entire supply chain network for the units. Next, a mathematical model is used to determine the optimal conditions for the merger of industrial units. After identifying the units that have to merge to achieve efficiency improvement, inverse DEA is used to determine the merged units' optimal inputs levels for achieving the target efficiency level. The proposed structure of the problem is shown in Figure 1.

3.1. DEA and Network DEA

Data Envelopment Analysis (DEA) is a mathematical programming model for evaluating the efficiency of decision-making units (DMUs) with multiple inputs and multiple outputs. This method was originally developed by Farrell for measuring the efficiency of production units [19]. Later, Charnes, Cooper, and Rhodes developed the DEA model for measuring the efficiency of units with multiple inputs and outputs under the assumption of a constant returns-to-scale (RTS). This model became known as the CCR model [12]. In 1984, Banker, Charnes, and Cooper generalized the CCR method for variable RTS, developing the model known as the BCC model [10]. DEA is one of the most widely used mathematical programming methods for evaluating and comparing the performance of a set of decision-making units (DMUs) in applications such as supplier selection. This model

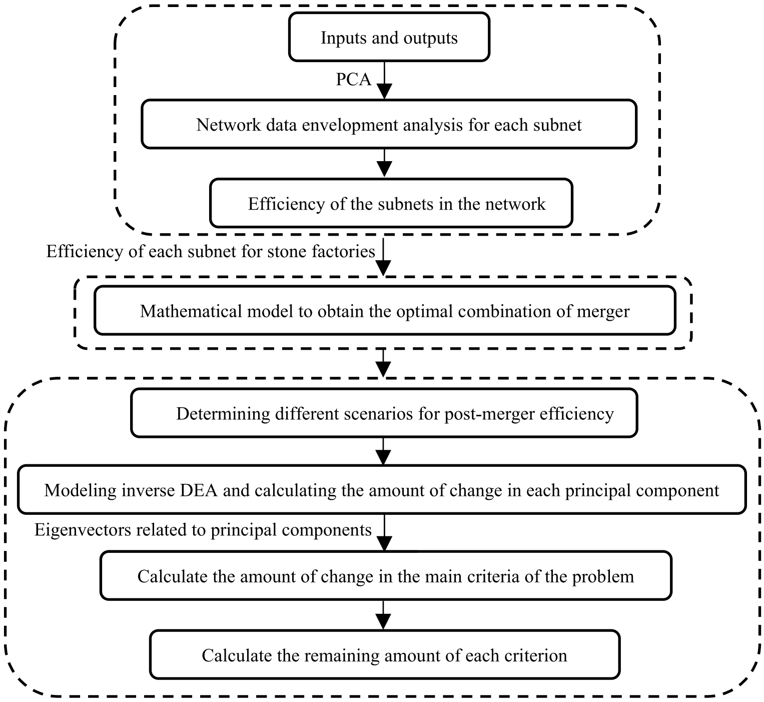

FIGURE 1. The proposed structure of the problem.

is known to maintain a good level of performance even when the analysis involves a large number of DMUs. For this reason, this study uses the DEA method for its evaluations.

The first DEA model proposed by Charnes *et al.* is in the form of equations (1) to (4). Assuming that there are n DMUs ($\text{DMU}_j, j = 1, 2, \ldots, n$) each with m inputs ($x_{ij}, i = 1, \ldots, m$) with weights v_i and s outputs ($y_{rj}, r = 1, \ldots, s$) with weights u_r, the relative efficiency of DMU_o is given by:

$$\text{Max} \sum_{r=1}^{s} u_r y_{ro} \tag{1}$$

$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0, \qquad j = 1, \ldots, n \tag{2}$$

$$\sum_{i=1}^{m} v_i x_{io} = 1 \tag{3}$$

$$u_r > 0, v_i > 0, \qquad r = 1, \ldots, s, i = 1, \ldots, m \tag{4}$$

where x_{io} and y_{ro} are the i-th input and r-th output of DMU_o respectively.

In this study, the criteria for the evaluation of industrial units and supply chain components and subnetwork are chosen from among the list of criteria previously used in other articles. These criteria, which also include sustainability criteria, are selected using the SCOR model. As mentioned, this study makes use of the network DEA approach for its evaluations. The general network layout considered in this study is illustrated in Figure 2. The mathematical model of the network DEA is presented in other sections.

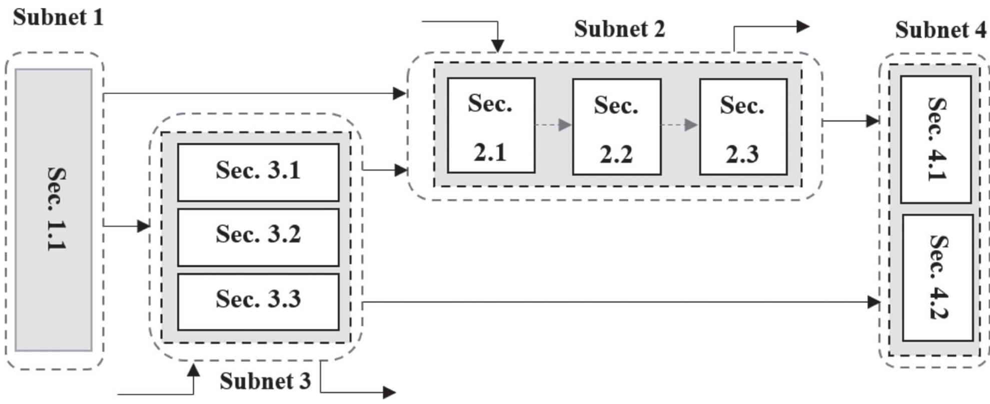

FIGURE 2. General network layout of the supply chain of industrial units.

3.2. Principal component analysis

In DEA, for efficiency evaluations to be reliable, the number of DMUs under evaluation must be at least three times the number of input and output criteria [10]. Considering the large number of criteria considered in this study for supply chain network evaluations, which also requires examining the conditions of a large number of industrial units, Principal Component Analysis (PCA) is used to reduce the dimensionality of the data while preserving the majority of the information contained in the data in order to make sure that the proposed model can be properly implemented with fewer DMUs. For a problem with n variables, the principal component i is expressed as:

$$\mathrm{PC}_i = a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{ij}x_j. \tag{5}$$

In the above equation, $i = 1, \ldots, n$ denotes the principal components and $j = 1, \ldots, n$ denotes the main problem variables (one can define as many principal components as there are problem variables). Also, a_{ij} is the j-th component of the linear transformation vector or eigenvector derived from the correlation matrix, and x_j is the j-th main variable. In this study, the PCA module provided in MATLAB was used to analyze the input and output criteria and choose the principal components that can preserve a high percentage of the total variance in the data for use in the NDEA model.

3.3. Mathematical model of the merger of industrial units

After determining the efficiency of industrial units, a mathematical model was developed for the merger of these units with the purpose of increasing their interactions, enabling them to utilize each other's capacities to increase efficiency, and achieving cost reduction (*e.g.* by reducing redundant labor costs). In this model, the goal is to find the best combinations for the binary integration of units with the objective of cost reduction considering the units' capacities and limitations. This model also assumes that the units to be merged must be in the same location. The parameters and variables of this mathematical model are as follows:

The mathematical model for the merger of units is given in equations (6) to (23).

$$\text{Max } Z = Nu \tag{6}$$

$$+\sum_i \sum_{j>i} \left[\left(\frac{D_i}{\rho} * Ccoup_i + \frac{D_j}{\rho} * Ccoup_j \right) - \left(\frac{D_i + D_j}{\rho} \right) * Ccoup_{i+j} * (1-\beta) \right] * f_{ij} \tag{7}$$

$$-\sum_i \sum_{j>i} \left[\left(\frac{Cap_i + Cap_j}{Cap_{hm}} \right) - (NumHm_i + NumHm_j) \right] * C_{hm} * f_{ij} * (1 - y_{ij}) \tag{8}$$

Parameters			
C_{hm}	Cost of hiring labor	I, j, p, q	Index of industrial units
Tax	Tax rate	L, k	Index of locations (to show the limitation of integration related to the difference in location of factories)
α_{in}^{i}	Percentage of domestic sales for industrial unit i	D_i	Demand allocated to industrial unit i
$Ccoup_i$	Unit price of product (1 tone of stone) in industrial unit i	Cap_i	Production capacity of industrial unit i
S_{it}	Inefficiency of industrial unit i in subnetwork t	$MinS_t$	Maximum inefficiency allowed for participation in national and international markets in subnetwork t
ρ	Square meter to tone conversion ratio	β	Percentage discount offered for large purchases
$NumHm_i$	Number of labor units in industrial unit i	Cap_{hm}	Production capacity of each labor unit
$il(i,l)$	The parameter indicating the location of each of the industrial units, which takes a value of one if the factory i belongs to the city l, otherwise it takes a value of zero		
Variables			
f_{ij}	$\begin{cases} 1 & \text{If unit } i \text{ is merged with unit } j \\ 0 & \text{Otherwise} \end{cases}$	y_{ij}	Binary variable denoting whether constraints are satisfied

$$-\sum_{i}\sum_{j>i}\Big[(Cap_i - D_i) * \alpha_{in}^{i} + (Cap_j - D_j) * \alpha_{in}^{j}\Big] * \text{Tax} * f_{ij} \tag{9}$$

$$(NumHm_i + NumHm_j) - \left(\frac{Cap_i + Cap_j}{Cap_{hm}}\right) \leq M * y_{ij} \qquad \forall i,j \tag{10}$$

$$(NumHm_i + NumHm_j) - \left(\frac{Cap_i + Cap_j}{Cap_{hm}}\right) \geq -M * (1 - y_{ij}) \qquad \forall i,j \tag{11}$$

$$Nu \leq \left[(NumHm_i + NumHm_j) - \left(\frac{Cap_i + Cap_j}{Cap_{hm}}\right)\right] * f_{ij} * y_{ij} \qquad \forall i,j,l,k \tag{12}$$

$$Ccoup_{i+j} = \min(Ccoup_i, Ccoup_j) \qquad \forall i,j \tag{13}$$

$$\frac{1}{2}(S_{it} + S_{jt}) \leq MinS_t + M * (1 - f_{ij}) \qquad \forall i,j,t \tag{14}$$

$$\sum_{j} f_{ij} * il(i,l) * il(j,k) \leq 1 \qquad \forall i,j,l,k,l=k,j>i \tag{15}$$

$$\sum_{i} f_{ij} * il(i,l) * il(j,k) \leq 1 \qquad \forall i,j,l,k,l=k,j>i \tag{16}$$

$$f_{ij} * il(i,l) * il(j,k) = 0 \qquad \forall i,j,l,k,l \neq k,j>i \tag{17}$$

$$f_{ij} = 0 \qquad \forall i,j,i=j \tag{18}$$

$$2 * f_{ij} + f_{pi} + f_{jq} \leq 2 \qquad \forall i,j,p,q \tag{19}$$

$$\sum_{j} f_{ij} \leq 1 \qquad \forall i \tag{20}$$

$$\sum_{i} f_{ij} \leq 1 \qquad \forall j \tag{21}$$

$$f_{ij} \in \{0,1\} \qquad \forall i,j \tag{22}$$

$$y_{ij} \in \{0,1\} \qquad \forall i,j. \tag{23}$$

In the above mathematical model, equation (6) computes the profit to be earned from the reduction of labor costs after the merger. In this equation, it is assumed that after the merger, all the capacities of the industrial units are used. So, the number of labor units after the merger is obtained by dividing the total capacity of two units by the average production capacity of each person. Since the goal is to maximize the difference between costs before and after the merger and the max–min composition is commonly used when a system requires conservative solutions, we have:

$$\text{Max Min}\left\{(NumHm_i + NumHm_j) - \left(\frac{Cap_i + Cap_j}{Cap_{hm}}\right) * f_{ij}, \quad \forall i,j\right\}. \tag{24}$$

Since using the above model makes the problem nonlinear, the parameter Nu is defined as follows:

$$Nu = \text{Min}\left\{(NumHm_i + NumHm_j) - \left(\frac{Cap_i + Cap_j}{Cap_{hm}}\right) * f_{ij}, \quad \forall i,j\right\}. \tag{25}$$

By substituting this parameter into the first term of the objective function and adding the constraint to equation (25), the problem becomes linearized as shown below:

$$\text{Max } Nu \tag{26}$$

$$Nu \leq (NumHm_i + NumHm_j) - \left(\frac{Cap_i + Cap_j}{Cap_{hm}}\right) * f_{ij}. \tag{27}$$

This constraint only applies if there are fewer labor units after the merger than before the merger. In other words, Industrial units will get this profit only if the number of labor units decreases after the merger. Otherwise, this profit amount will be zero. With the increase in the number of human resources, costs such as salary increases will be added, which is considered in equation (7). Therefore, the right side of constraint (27) is multiplied by the variable y_{ij} so that it gets activated whenever the mentioned condition applies. Otherwise, $y_{ij} = 0$ makes sure that the right side of constraint (27) remains zero, which results in Nu becoming zero as well. Thus, the constraint added to the model is in the form of equation (11). As mentioned earlier, equation (7) in the objective function is the profit to be earned from the reduction of procurement costs after the merger. Since the volume of procurements is expected to increase after the merger (assuming that more procurement will be made in the unit with lower procurement cost, as expressed in Eq. (13)), a volume discount is considered in the model.

Equation (8) expresses the recruitment costs after the merger. In some mergers, units need to hire more people in order to make full use of their new capacities, in which case the cost gets a negative sign in the model. Equation (9) computes the increase in taxes because of the increase in sales after the merger. In the case of the stone industry, this tax cost only applies to domestic sales because the export of building stones is exempt from tax.

Equations (10) and (11) determine the value of variable y_{ij}. If the number of labor units before the merger is greater than after the merger, then the parameter Nu must become non-zero and the recruitment costs should not be applied, and therefore the variable y_{ij} will be equal to one. Conversely, if the merger causes an increase in labor, the recruitment costs should be applied and the parameter Nu should be zero, and therefore y_{ij} will be zero. Equation (14) determines the maximum inefficiency required for the merger. In this equation, the average level of inefficiency for each sub-network and each decision unit has been compared with the maximum acceptable level for total inefficiency. This constraint only applies if the two units are merged ($f_{ij} = 1$) and is otherwise removed from the model. Equations (15) and (16) consider the reasonability of the merger. Only the units that are located in the same city can be merged, and each unit can only be merged with one other unit. Equation (17) indicates that the units located in different cities cannot be merged ($f_{ij} = 0$). Equation (18) indicates that a unit cannot be merged with itself. Equation (19) states that if unit i is merged with unit j, neither of them can then be merged with another unit like p or q. Thus, we have:

$$f_{ij} + f_{pi} \leq \text{۱}, \qquad \forall i,j,p,i \tag{28}$$

$$f_{ij} + f_{jq} \leq \text{۱}, \qquad \forall i,j,p,i. \tag{29}$$

These two constraints can be combined together and placed in the model as one constraint. Equations (20) and (21) of the model make sure that in the matrix of binary mergers, the variables f_{ij} can be non-zero at most once in each row and each column (meaning that each unit can only be merged with one unit in the same city). Equations (22) and (23) define the type of variables used in the model, which are all binary.

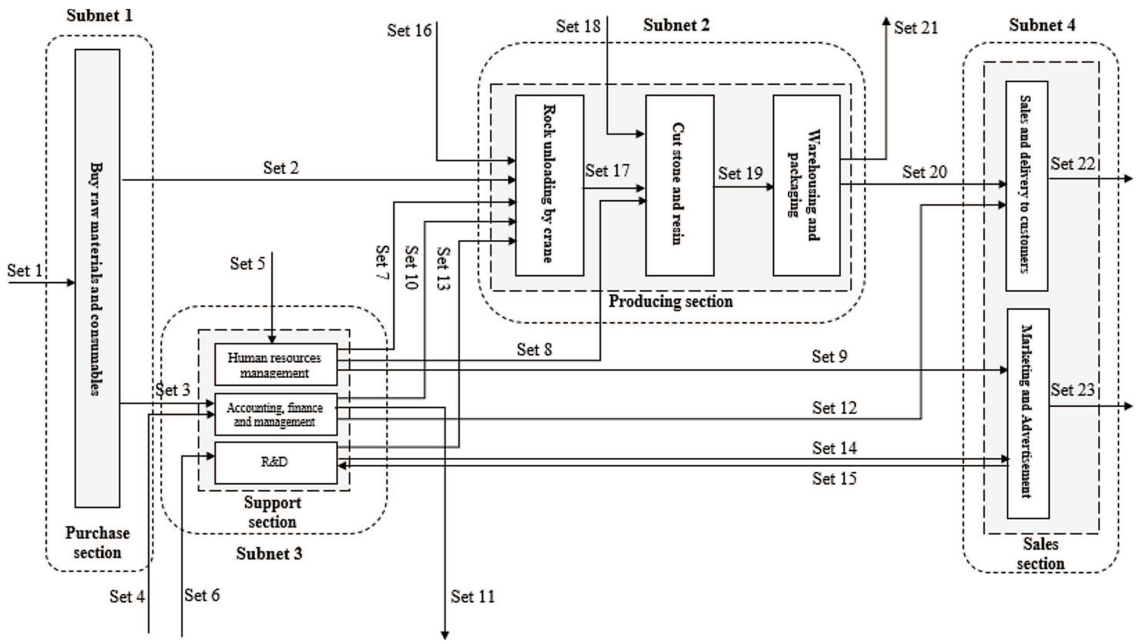

FIGURE 3. Network structure of the supply chain of the stone industry.

3.4. Inverse NDEA for the merged units

After determining the optimal combination for the merger, it is necessary to determine the units' post-merger inputs. Here, this was done using the inverse NDEA method. In the approach of inverse data envelopment analysis, an attempt is made to determine the optimal amount for inputs and outputs in order to achieve a certain efficiency. Since in this article, the aim is to determine the optimal combination for the integration of factories, it should be determined how much these units should maintain their criteria after the integration in order to achieve the desired efficiency. The best approach for this purpose is inverse data envelopment analysis. To develop the inverse NDEA model, first, the dual of DEA models had to be obtained. In this work, the mathematical model of inverse DEA was developed using the criteria considered for the case study, which are discussed in the next section, because mathematical models depend on the conditions of the case study.

4. MODEL IMPLEMENTATION

As mentioned, in this study, evaluations with the NDEA approach are performed based on the real data reported or used in other articles. The supply chain network of the considered factories is shown in Figure 3. Considering the sequence of activities in the selected industry and their prerequisite and dependency relations, the supply chain of this industry can be divided into four sub-networks. The first sub-network is related to the procurement of raw materials such as stone blocks and consumables such as resin, the second sub-network is related to support operations such as human resource management, accounting, finance, and R&D, the third sub-network is related to the production process, and finally, the fourth sub-network is related to sales and delivery.

After determining the supply chain network of the units and the relation between sub-networks, the criteria involved in the evaluation of sub-networks (inputs and outputs for each part of the network) were classified as shown in Table 1.

The efficiency of units was determined based on the real-world data used in other articles, which were collected by observation and face-to-face interviews. These data are provided in Table 2.

The results of the application of PCA to the above data in MATLAB are presented in Table 3. In this analysis, the principal components that were able to preserve the highest percentages of the total variance in the data were chosen for use in the rest of the study. The amount of variance preserved by these principal components is shown in Table 4.

Since DEA models cannot accept negative inputs, the following equations (30) and (31) were used to make all the figures related to principal components non-negative:

$$Z_j = \mathrm{PC}_j + Q \tag{30}$$

$$Q = -\min\{\mathrm{PC}_j\} + 1. \tag{31}$$

Also, since each principal component preserves one part of the total variance in the data, different components have different levels of importance for subsequent analyses. In other words, the greater the variance preserved by a component, the more valuable it will be. Therefore, the values of input and output criteria were entered into the DEA model based on the component's value. This value was calculated based on the ratio of the variance preserved by each individual component to the sum of the total variance covered by all of the selected components. First, the variance preserved by each component was used to compute a weight for that component in DEA. Then, the weight obtained for each criterion was multiplied by the non-negative values of that criterion in the respective column to obtain the weighted criteria values for use in the DEA model. The final results are presented in Table 5.

After preparing the data related to the inputs and outputs of the sub-networks of the supply chain, the CRS version of the DEA model was used to determine the efficiency of the stone processing units in all four sub-networks. The definitions of variables and parameters used in the model are provided in Table 6.

The DEA models developed for the raw material and consumable procurement department based on the CRS model are in the form of equations (32)–(35):

$$\text{Max} \ \sum_{L_1} v^1_{L_1} y^1_{L_1 k} \tag{32}$$

$$\sum_{I} s_I x_{Ik} = 1 \tag{33}$$

$$\sum_{L_1} v^1_{L_1} y^1_{L_1 j} - \sum_{I} s_I x_{Ij} \leq 0 \qquad \forall j \in \{1, 2, \ldots, N\} \tag{34}$$

$$v^1_{L_1}, s_I \geq \varepsilon \qquad \forall I \in \{1, 2, \ldots, i\}, \ \ \forall L_1 \in \{1, 2, \ldots, l_1\}. \tag{35}$$

The DEA models formulated for the support department based on the CRS model are provided in equations (36)–(42):

$$\text{Max} \ \sum_{F_1} z^1_{F_1 k} w^1_{F_1} + \sum_{F_2} z^2_{F_2 k} w^2_{F_2} + \sum_{F_3} z^3_{F_3 k} w^3_{F_3} \tag{36}$$

$$\sum_{L_2} y^2_{L_2 k} v^2_{L_2} + \sum_{L_3} y^3_{L_3 k} v^3_{L_3} + \sum_{L_4} y^4_{L_4 k} v^4_{L_4} = 1 \tag{37}$$

$$\sum_{F_1} z^1_{F_1 j} w^1_{F_1} + \sum_{F_2} z^2_{F_2 j} w^2_{F_2} + \sum_{F_3} z^3_{F_3 j} w^3_{F_3} - \left[\sum_{L_2} y^2_{L_2 k} v^2_{L_2} + \sum_{L_3} y^3_{L_3 k} v^3_{L_3} + \sum_{L_4} y^4_{L_4 k} v^4_{L_4}\right] \leq 0 \qquad \forall j \in \{1, 2, \ldots, N\} \tag{38}$$

$$\sum_{F_1} z^1_{F_1 j} w^1_{F_1} - \sum_{L_2} y^2_{L_2 j} v^2_{L_2} \leq 0 \qquad \forall j \in \{1, 2, \ldots, N\} \tag{39}$$

$$\sum_{F_2} z^2_{F_2 j} w^2_{F_2} - \sum_{L_3} y^3_{L_3 j} v^3_{L_3} \leq 0 \qquad \forall j \in \{1,2,\ldots,N\} \tag{40}$$

$$\sum_{F_3} z^3_{F_3 j} w^3_{F_3} - \sum_{L_4} y^4_{L_4 j} v^4_{L_4} \leq 0 \qquad \forall j \in \{1,2,\ldots,N\} \tag{41}$$

$$v^2_{L_2}, v^3_{L_3}, v^4_{L_4}, w^1_{F_1}, w^2_{F_2}, w^3_{F_3} \geq \varepsilon \qquad \forall L_2 \in \{1,2,\ldots,l_2\},\quad \forall L_3 \in \{1,2,\ldots,l_3\}, \quad \forall L_4 \in \{1,2,\ldots,l_4\},\quad \forall F_1 \in \{1,2,\ldots,f_1\}. \tag{42}$$

The DEA models for the manufacturing department based on the CRS model are in the form of equations (43) to (49):

$$\text{Max } \sum_{U_5} T^1_{U_5 k} H^1_{U_5} \tag{43}$$

$$\sum_{U_1} z^4_{U_1 k} w^4_{U_1} + \sum_{U_3} z^6_{U_3 k} w^6_{U_3} = 1 \tag{44}$$

$$\sum_{U_5} T^1_{U_5 j} H^1_{U_5} - \left[\sum_{U_1} z^4_{U_1 j} w^4_{U_1} + \sum_{U_3} z^6_{U_3 j} w^6_{U_3}\right] \leq 0 \qquad \forall j \in \{1,2,\ldots,N\} \tag{45}$$

$$\sum_{U_2} z^5_{U_2 j} w^5_{U_2} - \sum_{U_1} z^4_{U_1 j} w^4_{U_1} \leq 0 \qquad \forall j \in \{1,2,\ldots,N\} \tag{46}$$

$$\sum_{U_4} z^7_{U_4 j} w^7_{U_4} - \left[\sum_{U_2} z^5_{U_2 j} w^5_{U_2} + \sum_{U_3} z^6_{U_3 j} w^6_{U_3}\right] \leq 0 \qquad \forall j \in \{1,2,\ldots,N\} \tag{47}$$

$$\sum_{U_5} T^1_{U_5 j} H^1_{U_5} - \sum_{U_4} z^7_{U_4 j} w^7_{U_4} \leq 0 \qquad \forall j \in \{1,2,\ldots,N\} \tag{48}$$

$$w^4_{U_1}, w^5_{U_2}, w^6_{U_3}, w^7_{U_4}, H^1_{U_5} \geq \varepsilon \qquad \forall U_1 \in \{1,2,\ldots,u_1\},\quad \forall U_2 \in \{1,2,\ldots,u_2\}, \quad \forall U_3 \in \{1,2,\ldots,u_3\},\quad \forall U_4 \in \{1,2,\ldots,u_4\}, \quad \forall U_5 \in \{1,2,\ldots,u_5\}. \tag{49}$$

The DEA models developed for the sales department based on the CRS model are given in equations (50) to (55):

$$\text{Max } \sum_{S_1} P^1_{S_1 k} E^1_{S_1} + \sum_{S_2} P^2_{S_2 k} E^2_{S_2} \tag{50}$$

$$\sum_{R_2} T^2_{R_2 k} H^2_{R_2} + \sum_{R_3} T^3_{R_3 k} H^3_{R_3} = 1 \tag{51}$$

$$\sum_{S_1} P^1_{S_1 j} E^1_{S_1} + \sum_{S_2} P^2_{S_2 j} E^2_{S_2} - \left[\sum_{R_2} T^2_{R_2 j} H^2_{R_2} + \sum_{R_3} T^3_{R_3 j} H^3_{R_3}\right] \leq 0 \qquad \forall j \in \{1,2,\ldots,N\} \tag{52}$$

$$\sum_{S_1} P^1_{S_1 j} E^1_{S_1} - \sum_{R_2} T^2_{R_2 j} H^2_{R_2} \leq 0 \qquad \forall j \in \{1,2,\ldots,N\} \tag{53}$$

$$\sum_{S_2} P^2_{S_2 j} E^2_{S_2} - \sum_{R_3} T^3_{R_3 j} H^3_{R_3} \leq 0 \qquad \forall j \in \{1,2,\ldots,N\} \tag{54}$$

$$H^2_{R_2}, H^3_{R_3}, E^1_{S_1}, E^2_{S_2} \geq \varepsilon \qquad \forall R_2 \in \{1,2,\ldots,r_2\},\quad \forall R_3 \in \{1,2,\ldots,r_3\}, \quad \forall S_1 \in \{1,2,\ldots,s_1\},\quad \forall S_2 \in \{1,2,\ldots,s_2\}. \tag{55}$$

The results of the implementation of the four DEA models in GAMS software to determine the efficiency of each department are shown in Table 7.

After determining the efficiency of each unit, the mathematical model for the merger was applied to the case study data. In this model, the goal is to determine the best combinations for the binary merger of units with the objective of cost reduction considering the units' capacities and the defined constraints. The results of model implementation in GAMS showed that there should be a merger between units 1 and 5, units 10 and 26, units 12 and 15, units 13 and 22, units 14 and 19, and units 23 and 24.

Table 1. Classification of criteria for the performance evaluation of units based on the combination of BSC and SCOR.

Type of criteria		Description of criteria	Type of criteria		Description of criteria
Purchase inputs	S1-1	Access to stone mines	Human R.M. inputs & Adv. inputs	S9-1	Familiarity with the science of stone marketing
	S1-2	Access to consumables (*e.g.* resin)	Finance outputs & Unloading inputs	S10-1	Salary costs per person per hour
	S1-3	Time required to replace machinery and equipment		S10-2	Need for the monitoring of production line
	S1-4	Time required to switch to a new quarry in an emergency	Finance outputs	S11-1	Liquidity available in the unit (representing the unit's ability to address risks)
	S1-5	Average distance between stone quarries and processing units	Finance outputs & Delivery inputs	S12-1	Share of administrative costs in the total product cost
Purchase outputs & Unloading inputs	S2-1	Percentage of waste produced per tones of stone	R&D outputs & Unloading inputs	S13-1	Speed of adopting new stone cutting and processing techniques
	S2-2	Amount of mud produced per tones of stone	R&D outputs & Adv. Inputs	S14-1	Ability to meet the diverse needs of customers
Purchase outputs & Finance inputs	S3-1	Share of raw materials and consumables in the total product cost		S14-2	Share of marketing costs in the total product cost
	S3-2	Cost of raw materials (*e.g.* stone blocks and consumables)	Avs. Outputs & R&D inputs	S15-1	Feedback from customers
	S3-3	Transport cost per unit volume of stone per kilometer		S15-2	Knowledge of the target markets
	S3-4	Financial stability of stone quarries affiliated with the processing unit	Unloading inputs	S16-1	Maintenance and repair cost per period
Finance inputs	S4-1	Percentage of checks cleared and proceeds received		S16-2	Amount of electricity consumed per period in normal time
Human resource management inputs	S5-1	Number of employees in the non-production parts of the business	Unloading outputs & Cutting inputs	S17-1	Precautionary storage of raw materials and consumables (*e.g.* resin)
	S5-2	Number of employees in the production part of the business	Cutting inputs	S18-1	Average machinery repair time
	S5-3	Number of hours of formal training		S18-2	Use of machinery after the end of their life
	S5-4	Number of hours of informal and on-the-job training		S18-3	Amount of water consumed by each machinery per period
	S5-5	Management's training level	Cutting outputs & Warehousing inputs	S19-1	Speed of cutting and processing of stone into the final product
	S5-6	Employees' safety training level		S19-2	Difference between the unit's nominal capacity and actual production
R&D inputs	S6-1	Unit's land area	Warehousing outputs & Delivery inputs	S20-1	Percentage of work stoppage in each period
	S6-2	Familiarity with international trade rules for exporting stone		S20-2	Cost of transporting one unit volume of product from the unit to the customer per kilometer
Human resource management outputs & Unloading inputs	S7-1	Number of shifts and working hours		S20-3	Ratio of exports to total production
	S7-2	Ratio of native workers to non-native workers		S20-4	Share of production costs in the total product cost
	S7-3	Ratio of non-native workers without work license	Warehousing outputs	S21-1	Amount of product stored at the end of each period
	S7-4	Fines paid due to unlicensed non-native workers	Delivery outputs	S22-1	Time required to meet customer needs
	S7-5	Safe environment for employees		S22-2	Direct sales to consumers
	S7-6	Employees' exposure to environmental hazards and pollution		S22-3	Sales to brokers and intermediaries
Human resource management outputs & Cutting inputs	S8-1	Level of expertise of human resources based on their degree	Adv. outputs	S23-1	Percentage change in sales due to marketing and advertising
	S8-2	Level of work experience of human resources			

TABLE 2. Data used in the NDEA model.

Factors	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10	DMU11	DMU12	DMU13	DMU14	DMU15	DMU16	DMU17	DMU18	DMU19	DMU20	DMU21	DMU22	DMU23	DMU24	DMU25	DMU26
S1-1	2	1	1	1.5	1	1	0.5	0.5	1	0.7	2	3	0.4	1	0.5	0.6	0.5	1	1	2	2	0.3	1	1	1.5	0.4
S1-2	1	1	1	0	2	3	0	1	2	4	1	2	1	1.5	1	1	1	2	1	2	3	1	1	2	1	1
S1-3	1	0.5	0.5	1	4	8	1	1	2	1.5	4	10	5	4	3	4	2	2	3	6	4	4	2	8	0.4	2
S1-4	0.5	2	0.5	0.8	6	0.5	5	0	1	1	0.1	0.5	0.2	0.5	1	2	1	1	3	2	2	1	2	2	0.5	1
S1-5	200	500	700	300	300	400	135	100	500	250	350	200	400	380	200	180	140	140	600	150	160	400	300	550	550	350
S2-1	30	50	50	35	35	30	28	25	30	25	30	35	30	30	30	35	40	40	35	50	50	30	25	35	40	30
S2-2	150	250	500	100	130	100	50	130	80	200	120	150	50	70	60	60	50	40	50	40	35	60	150	50	180	50
S3-1	90	80	80	70	75	70	80	70	85	70	75	60	70	70	70	60	55	50	70	60	60	75	90	80	75	70
S3-2	900	950	800	750	450	250	300	950	500	480	300	500	700	480	700	300	200	200	500	300	200	800	1200	800	700	800
S3-3	500	750	500	300	250	350	400	225	340	220	200	300	300	350	350	350	220	250	600	250	300	400	350	600	550	400
S3-6	15 000	20 000	500	1500	3500	7000	1300	900	1300	850	1600	500	4000	2000	4500	3400	950	1400	8000	4500	890	9000	20 000	4700	900	830
S4-1	80	90	90	80	100	70	75	50	70	70	60	70	80	85	90	85	75	70	80	70	75	85	80	70	80	70
S5-1	4	15	5	8	4	2	1	2	4	4	4	2	4	2	2	1	1	1	2	1	2	3	4	2	6	3
S5-2	19	40	13	15	20	10	11	15	10	13	10	8	15	10	7	3	4	2	5	4	3	8	20	8	15	9
S5-3	0	10	0	5	7	0	0	1	0	0	5	0	2	0	2	0	0	0	0	0	0	2	4	1	0	6
S5-4	2	5	0	5	10	10	5	10	10	15	8	5	10	5	4	1	2	3	1	1.5	2	5	7	7	0	3
S5-5	2	1	2	2	2	1	3	2	3	2	2	2	1	3	3	2	1	3	3	3	3	2	3	2	3	2
S5-6	4	3	4	4	5	3	3	4	2	3	2	4	1	2	3	4	5	5	3	3	2	2	3	4	4	4
S6-1	5	8	5	7	6.5	11	6.5	6	6	5	3.8	6	7	4.5	4	3.8	3.5	3.7	4	3.9	4.2	6	11	10	5	5
S6-2	3	1	1	3	5	1	3	1	2	1	1	1	2	1	3	2	2	2	2	1	1	3	4	3	2	2
S7-1	10	12	11	8	12	10	10	12	10	8	8	8	8	10	12	10	12	8	12	8	8	10	20	12	12	12
S7-2	95	65	20	100	90	20	50	15	50	40	65	60	50	80	90	100	100	100	100	90	85	90	50	100	30	70
S7-3	0	35	13	0	0	8	3	8	19	0	5	8	10	5	0	2	2	1	1	2	2	0	2	3	10	2
S7-4	0	0	0	0	0	5	0	6	0	0	0	0	5	5	5	0	0	6	0	0	4	0	0	5	0	0
S7-5	3	4	3	4	4	3	3	4	2	3	3	4	4	3	4	3	3	3	4	3	3	4	4	4	3	4
S7-6	3	3	2	4	3	4	3	4	4	3	4	4	3	3	3	3	3	3	3	3	3	3	3	3	3	2
S8-1	3	2	1	2	3	3	3	1	2	3	?	2	3	1	2	2	2	2	2	2	2	2	3	2	2	2
S8-2	8	5	10	2	15	15	10	8	7	5	6	10	15	8	15	6	3	2	7	4	4	10	11	15	10	12
S9-1	3	1	1	3	5	3	3	1	1	4	3	3	4	2	3	1	1	1	2	1	1	3	3	3	1	3
S10-1	9000	10 000	8000	7500	7500	6000	10 000	5000	8000	9500	7000	9000	12 000	10 000	15 000	10 000	11 000	10 000	14 000	12 000	12 000	15 000	17 000	14 000	8000	13 000
S10-2	90	100	100	70	50	80	100	100	100	80	100	80	100	80	100	50	40	60	80	60	70	100	90	85	100	100
S11-1	0	500	500	500	200	300	200	700	200	300	400	0	500	200	100	200	0	0	50	150	0	100	200	70	500	100
S12-1	2.5	10	10	10	10	10	7	10	5	10	5	10	5	10	15	10	10	5	15	10	10	10	2.5	5	10	10
S13-1	4	3	1	3	3	1	1	5	1	3	3	2	4	2	1	1	2	2	2	2	2	3	4	2	1	2
S14-1	15	12	30	15	30	90	90	60	14	7	3	7	4	10	14	10	11	9	11	12	15	10	14	7	20	12
S14-2	2.5	0	5	5	5	0	3	0	0	0	5	10	5	10	5	10	0	0	0	0	0	5	2.5	0	5	10
S15-1	5	3	4	4	4	3	3	4	2	3	4	4	3	2	5	4	4	1	3	3	2	3	4	3	2	3
S15-2	3	1	1	3	4	3	3	4	4	4	2	2	4	3	3	1	1	1	1	1	2	2	3	3	2	3
S16-1	4	6	3.5	4	7.5	5	8	12	5.5	20	11	7	6	4	5.5	3.5	2.5	2.5	4.5	2.5	2	7	11	4	6	4.5
S16-2	10	25	15	12	6	3.5	8	5.5	4.5	3	9	8.7	7.8	8	3.2	2	1.9	1.5	2.5	1.7	1.4	3.5	10	3	15	3.8
S17-1	2	3	0	2	2	1.5	2	1.5	0	2	3	0	2	2	1.5	1.75	1.5	2.5	2	2	1.5	1.5	1.25	2	0	2
S18-1	2	48	2	24	0	48	24	48	12	12	48	24	12	48	24	24	36	48	36	48	48	24	48	48	48	24
S18-2	1	2	2	1	3	2	2	1	1	1	2	4	1	1	2	1	2	2	2	3	3	2	1	2	3	1
S18-3	3	4	2.8	3	8	3	3	3.5	0.5	3	4	4.5	2.9	3	3	4	5	5.3	4.8	3.5	6	4	4	3.8	2.8	4
S19-1	40	22	25	22	20	12	10	17	15	15	20	12.5	37	20	11	12	12	10	11	9	8	15	35	14	22	12.5
S19-2	5	0	4	0	3	0	0.5	3	4.5	2.4	4	3	1	2	2	3	2.5	3	2.5	2	3.5	3	2	2	4	2.5
S20-1	0	0	2	7	0	1	2	0	4	0	3	5	0	0	0	2	7	0	1	2	0	4	0	3	5	0
S20-2	25	35	55	30	15	10	25	30	30	28	18	22	30	25	35	55	30	15	10	25	30	30	28	18	22	30
S20-3	15	30	0	0	90	0	0	0	0	0	0	0	0	15	30	0	0	90	0	0	0	0	0	0	0	0
S20-4	5	10	5	15	10	20	10	20	10	20	15	20	20	5	10	5	15	10	20	10	20	10	20	15	20	20
S21-1	2000	2000	1800	2000	1500	3000	3000	1300	1000	1000	2000	1500	2000	2000	2000	500	200	1000	1500	500	500	2500	4000	3000	2000	3500
S22-1	1	1	2	5	45	15	3	3	1	1	3	7	5	4	2	2	3	2	2	4	5	2	1	3	2	2
S22-2	2	1.6	2	4	5	3.2	2.1	1.8	4.5	3.24	2.8	1.5	5	3	1.2	2	0.4	0.5	0.8	0.6	0.2	1.5	5	2	2.5	1.5
S22-3	8	6.4	5.5	1.2	1	0.8	0.9	6.4	0	0.36	2	1.5	4	1.8	1.5	0.8	1.6	1.5	1.2	0.9	1.1	2	2	1.5	5.5	1.5
S23-1	70	50	40	40	30	60	30	20	80	45	40	50	90	30	75	50	40	50	20	35	50	40	60	70	40	30

After obtaining the optimal combinations for the merger, the units' post-merger inputs were determined using the inverse NDEA method. To develop the inverse NDEA model, first, the dual of DEA models had to be obtained. The dual of the CRS-type DEA model for raw material and consumable procurement department are as follows:

$$\text{Min } \theta_{\text{Purchase}} \tag{56}$$

$$\sum_j \lambda_j x_{Ij} \leq \theta_{\text{Purchase}} * x_{Ik} \qquad \forall I \in \{1, 2, \ldots, i\} \tag{57}$$

$$\sum_j \lambda_j y^1_{L_1 j} \geq y^1_{L_1 k} \qquad \forall L_1 \in \{1, 2, \ldots, l_1\} \tag{58}$$

$$\lambda_j \geq 0 \qquad \forall j \in \{1, 2, \ldots, N\}. \tag{59}$$

TABLE 3. Selected principal components.

DMUs	I_a			O_a	I_b		O_b		I_c	O_c		I_d	O_d	I_e	O_e		I_f		O_f		I_g		Mo_g, I_h	I_h	Mo_h, I_i	O_i	
	1	2	3	1	1	2	1	2	1	1	2	1	1	1	1	2	1	2	1	2	1	2	1	1	1	1	2
DMU1	−2.09787	−1.03403	−1.36132	11.1236	2.96814	1.68478	−1.49044	2.13983	−111.139	3.24877	5.38637	0.37723	−0.81592	−11.6567	−5.66948	0.25072	0.68851	−0.64957	2.40226	0.58487	1.28941	−5.34567	0.3654	0.20691	−2.75643	2.38075	−0.85902
DMU2	−2.8322	1.47524	0.90321	16.2048	−8.84865	11.4662	−3.10982	−2.30007	−162.29	0.9031	−1.72628	−1.4802	−3.15078	−6.64057	−4.12	−0.31417	3.22271	−1.63927	−0.13377	−1.46517	32.2362	18.3684	1.3654	3.75277	2.41305	3.00483	0.41907
DMU3	−2.87397	3.79334	−0.03152	−2.76199	4.17315	3.4658	2.30606	−4.86041	27.9827	2.46531	−2.59602	1.31436	1.4262	−11.0848	−3.18249	−0.00847	−1.26297	1.13979	−1.04794	−0.83264	37.2711	−14.2082	−1.6346	−1.71428	−1.59547	−0.5264	−0.70544
DMU4	−2.35363	−0.11103	−0.73579	−2.45559	−3.21132	4.57506	−7.55126	1.32743	24.3436	2.51756	−3.06545	−1.47023	1.66506	−0.72441	1.28624	1.4865	−1.74572	−0.13698	−0.55355	0.94583	−1.84209	−2.1305	0.3654	6.45263	2.41305	−3.88071	2.2593
DMU5	0.75065	−1.38653	4.3605	−1.51857	−6.29579	0.09923	5.79477	3.57662	13.8839	1.14078	−2.56094	−1.63504	1.34424	−7.13102	1.9866	3.54669	−1.5905	1.50939	−1.27386	2.08343	−1.55061	−5.8902	0.3654	−6.71991	−0.54597	6.91684	5.11516
DMU6	4.91394	1.12863	−0.53355	1.21219	−2.39403	−4.65053	7.63841	−2.26104	−11.7637	3.03448	−3.18542	−4.82558	−4.73708	3.47965	1.71467	0.9614	4.71765	6.14571	1.32812	0.06493	−0.08369	2.36601	−0.1346	−6.22376	2.52451	0.83957	−1.44491
DMU7	−2.48395	−2.76847	2.96752	−3.81844	2.02889	−2.1778	1.54467	−1.55321	38.0792	1.19029	1.55049	−0.94997	−1.80908	−6.30375	1.39379	−0.41389	1.79657	6.76082	−1.57437	0.81004	−4.76174	1.39809	0.3654	−1.53621	2.04992	0.08898	−0.89987
DMU8	−2.20417	−1.83314	−1.97122	−2.48081	−2.77026	−4.08245	1.82228	−4.71281	26.5004	5.57611	−4.34897	−0.3889	−4.25936	3.44897	−4.08155	−0.06717	4.24345	3.03777	−2.24135	2.3318	3.6998	−0.33715	−0.1346	0.81733	−0.51253	0.91573	−1.77784
DMU9	−1.17862	1.76571	−0.03146	−3.2192	−3.14271	−3.53667	−1.51812	−2.94359	32.4894	3.71145	2.41693	−0.48862	−3.1116	−6.09462	2.47818	1.71157	3.26524	−1.4444	3.45699	0.07327	3.94118	11.8981	−1.6346	1.32026	−1.98089	−1.78271	0.52358
DMU10	−1.28476	−0.70588	−0.74712	−3.96816	−7.06214	−6.53577	−3.07525	−3.32683	40.3645	0.30062	−1.66338	0.6049	−3.04384	3.44091	2.00947	0.55308	2.87073	−1.84927	0.12407	1.32672	3.90052	−12.8099	0.3654	3.29914	0.10602	0.8985	−1.75419
DMU11	0.79741	0.6275	−1.22808	−3.82715	−3.90649	0.10768	−2.99693	−1.0303	39.2396	5.26005	1.53373	2.19881	1.92171	−1.30189	0.36632	0.37919	−2.0009	−1.30624	−0.80075	0.05659	2.09067	−1.34292	1.3654	2.72189	−1.53974	−0.79624	−0.1712
DMU12	6.9917	−0.74905	−1.07483	−4.25005	1.63337	−1.52814	0.77615	−0.50188	42.971	−0.46007	−1.20831	0.11521	6.75714	3.97316	0.78735	−0.80318	−6.78429	0.10869	0.16675	−0.19178	5.19999	−0.06171	−1.6346	−1.49906	−0.46237	−2.50504	1.14481
DMU13	1.59382	1.15874	−0.82613	−0.2829	−4.31402	−2.53736	6.51672	0.90854	2.50087	1.73828	3.48391	−1.48243	1.85934	3.365	−1.39868	2.76073	−2.06152	−1.11641	4.47781	0.20906	−2.10383	7.55146	0.3654	−6.62746	1.25208	1.03713	−1.85214

Legend. I: Inputs. O: Outputs. M: Middle. a: Purchase. b: Human resources. c: Accounting and finance. d: Research and development. e: Delivery to customers. f: Advertise. g: Drain the stone. h: Stone cutting. i: Store.

Table 3. continued.

DMUs	I_a			O_a	I_b		O_b		I_c	O_c		I_d	O_d	I_e	O_e		I_f		O_f		I_g		Mo_g, I_h	I_h	Mo_h, I_i	O_i	
	1	2	3	1	1	2	1	2	1	1	2	1	1	1	1	2	1	2	1	2	1	2	1	1	1	1	2
DMU14	0.75641	0.82447	−0.7048	−2.76521	1.56169	−1.63818	−0.50514	1.42094	27.0488	−0.47368	−1.22177	1.37722	6.69298	−6.2171	0.59257	0.57493	−6.63861	0.3086	−1.62769	0.42588	−3.35847	2.96635	0.3654	0.8109	0.4478	−0.24102	−0.75684
DMU15	−0.30067	−1.05414	−0.66428	0.25329	1.54104	−0.04487	6.26416	4.18999	−3.4045	−6.9348	−2.99572	1.32432	1.7684	−6.41806	0.70901	−1.23022	−1.76698	−0.23442	2.88601	0.46069	−8.73626	−1.45055	−0.1346	−6.46053	0.54812	1.62285	−1.58244
DMU16	0.67645	−1.47186	0.28399	−2.11742	5.21074	0.29552	−3.73175	1.76414	20.4929	−0.81198	−1.47376	2.25772	6.73396	3.75215	1.47363	−0.54888	−6.55673	0.2162	−0.08045	−1.08101	−8.12923	−0.21311	0.1154	2.49537	−0.4568	−1.0088	−0.42031
DMU17	−1.25929	−1.66983	−0.8623	−4.94525	4.40947	−0.23705	−5.94087	0.83778	49.2572	−2.47335	−0.67455	2.54185	−3.08841	18.6093	0.54912	−1.95228	3.20144	−1.73671	−1.04794	−0.83264	−9.06914	0.42333	−0.1346	5.61314	0.04009	−1.40477	−0.22083
DMU18	−1.04654	−1.70624	−0.94544	−4.5378	3.81423	−0.92825	−6.93141	2.09606	45.4417	0.93068	3.39902	2.49257	−3.04564	28.8672	0.64675	−1.89682	3.15891	−1.93159	−0.2404	−2.23348	−10.702	0.79049	0.8654	6.74708	−0.43451	−4.33624	2.17913
DMU19	−0.4384	2.27276	2.23299	3.22977	4.77983	0.84838	−2.14251	2.054	−32.6561	−6.5711	−3.4865	2.11501	−3.08841	−1.08558	0.97063	−1.64563	3.11956	−1.64431	−3.03626	−0.72006	−9.22982	−0.46872	0.3654	1.64044	0.05123	−1.09172	−0.06895
DMU20	2.95357	−1.74489	0.14117	−1.2321	4.82627	−0.082	−5.77133	0.5398	12.3725	−2.45329	−0.40794	2.40287	−3.1098	13.819	1.27071	−1.81404	3.22271	−1.63927	−1.58501	−1.09261	−9.83557	1.15666	0.3654	4.72862	0.57041	−2.41499	0.82065
DMU21	1.15023	−1.74706	0.03364	−4.85757	3.97259	0.14632	−5.02117	0.82198	48.281	−2.91615	0.0123	1.9134	−3.17396	14.1021	1.04733	−2.14368	3.2865	−1.34696	0.06011	−0.96009	−10.4444	1.80092	−0.1346	4.76078	−0.9091	−4.1138	2.07277
DMU22	0.58611	0.91649	−0.09417	4.89024	0.19887	−0.08163	0.47229	2.21983	−49.4356	−4.53741	1.34473	−0.22437	1.772	−6.44773	0.24036	−0.88705	−1.85204	−0.62418	−0.85407	−0.32756	−8.31161	−2.02023	−0.1346	−1.485	−0.49024	1.31834	−2.00877
DMU23	−1.28789	−0.39808	0.46398	16.5407	−2.68188	0.40708	5.01335	−2.16112	−165.2	−2.23255	8.41699	−5.45174	−0.79454	−11.8354	0.54132	2.41768	0.66725	−0.74701	1.38144	0.44909	2.86731	−5.65333	−0.3846	−2.23052	0.28061	4.59626	1.74638
DMU24	4.66572	2.21061	1.33131	1.16927	−0.35936	−2.29794	5.99399	5.06659	−11.4497	−1.64319	5.19158	−4.26479	−3.00286	−1.45327	0.79049	−0.44012	2.95261	−1.94167	2.29562	−0.18344	−8.37798	1.63127	0.3654	−6.17512	0.51468	0.5828	−0.0565
DMU25	−2.84748	1.84891	−0.58547	−2.77508	3.8028	4.07436	2.29887	−3.99858	27.7452	2.46531	−2.59602	0.96258	1.64008	−6.24128	−3.13802	0.46767	−1.47562	0.16541	−0.90739	−0.71172	11.6303	1.05377	−1.6346	−1.23557	−1.56203	−0.13664	−0.78143
DMU26	−1.34657	0.35785	−0.32082	−2.81049	0.06558	3.18823	3.34429	0.68631	28.3443	−2.9752	0.47499	0.66381	6.6502	−6.22113	0.73569	−0.94454	−6.67796	0.59588	−1.57437	0.81004	−7.59002	0.52728	0.3654	−3.45982	0.03451	0.03651	−0.92016

TABLE 4. Amount of variance preserved by the selected principal components.

	Procurement input	Procurement output	Human Res. input	Human Res. output	Financial Man. input	Financial Man. output	R&D input	R&D output	Offloading input	Offloading output	Cutting input	Cutting output	Packaging output	Delivery input	Delivery output	AD. input	AD. output
Number of Criteria	5	6	6	9	5	4	4	3	12	1	5	2	5	5	3	3	3
Number of PCAs	2	1	2	2	1	2	1	1	2	1	1	1	2	1	2	2	2
Covered Variance	0.758368	0.808358	0.883025	0.679027	0.819278	0.780974	0.632955	0.679688	0.776797	1	0.758984	0.716383	0.787809	0.840984	0.923713	0.933966	0.837137

TABLE 5. Non-negative and weighted values of the selected principal components.

DMUs	I_a			O_a	I_b		O_b		I_c	O_c		I_d	O_d	I_e	O_e		I_f		O_f		I_g		Mo_g, I_h	I_h	Mo_h, I_i	O_i	
	1	2	3	1	1	2	1	2	1	1	2	1	1	1	1	2	1	2	1	2	1	2	1	1	1	1	2
DMU1	1.0374	0.6603	0.2808	17.069	7.3492	3.9334	5.1950	2.1140	55.06	5.7435	5.2220	6.8290	4.9212	1.1786	0.6791	1.0893	6.1440	0.6300	4.8659	0.9326	10.038	2.2422	3.0000	7.9268	1.0000	5.4357	0.6355
DMU2	0.6085	1.2663	0.6758	22.150	0.5734	8.1061	4.0036	0.9408	3.91	4.5389	1.7622	4.9715	2.5863	6.1948	1.7313	0.9080	7.9816	0.3580	2.9493	0.4319	33.949	7.6334	4.0000	11.473	6.1695	5.8753	1.0133
DMU3	0.5841	1.8261	0.5128	3.1833	8.0402	4.6932	7.9883	0.2642	194.18	5.3412	1.3391	7.7661	7.1633	1.7505	2.3680	1.0061	4.7289	1.1218	2.2584	0.5864	37.839	0.2273	1.0000	6.0056	2.1610	3.3880	0.6809
DMU4	0.8880	0.8832	0.3899	3.4897	3.8059	5.1664	0.7358	1.8993	190.54	5.3680	1.1108	4.9815	7.4021	12.111	5.4026	1.4859	4.3788	0.7709	2.6320	1.0208	7.6183	2.9731	3.0000	14.173	6.1695	1.0253	1.5573
DMU5	2.7012	0.5752	1.2788	4.4267	2.0372	3.2570	10.555	2.4937	180.08	4.6609	1.3562	4.8167	7.0813	5.7044	5.8782	2.1470	4.4914	1.2234	2.0877	1.2987	7.8435	2.1184	3.0000	1.0000	3.2105	8.6309	2.4016
DMU6	5.1329	1.1826	0.4252	7.1574	4.2745	1.2308	11.912	0.9511	154.44	5.6335	1.0524	1.6262	1.0000	16.315	5.6936	1.3174	9.0657	2.4978	4.0541	0.8056	8.9769	3.9954	2.5000	1.4962	6.2809	4.3501	0.4623
DMU7	0.8119	0.2415	1.0359	2.1268	6.8107	2.2857	7.4281	1.1382	204.28	4.6863	3.3561	5.5018	3.9280	6.5316	5.4757	0.8760	6.9475	2.6668	1.8606	0.9876	5.3624	3.7753	3.0000	6.1837	5.8063	3.8214	0.6234
DMU8	0.9753	0.4674	0.1744	3.4644	4.0588	1.4732	7.6324	0.3032	192.70	6.9388	0.4864	6.0628	1.4777	16.284	1.7574	0.9873	8.7218	1.6435	1.3565	1.3593	11.900	3.3808	2.5000	8.5372	3.2439	4.4038	0.3639
DMU9	1.5743	1.3364	0.5128	2.7261	3.8452	1.7060	5.1747	0.7708	198.69	5.9811	3.7776	5.9631	2.6255	6.7408	6.2121	1.5581	8.0125	0.4115	5.6630	0.8077	12.088	6.1625	1.0000	9.0402	1.7755	2.5030	1.0442
DMU10	1.5123	0.7396	0.3879	1.9771	1.5978	0.4266	4.0290	0.6695	206.56	4.2294	1.7928	7.0566	2.6932	16.276	5.8938	1.1863	7.7264	0.3003	3.1442	1.1138	12.055	0.5452	3.0000	11.02	3.8625	4.3916	0.3709
DMU11	2.7285	1.0616	0.3041	2.1181	3.4073	3.2606	4.0866	1.2763	205.44	6.7764	3.3480	8.6505	7.6588	11.534	4.7779	1.1305	4.1938	0.4495	2.4452	0.8036	10.657	3.1522	4.0000	10.442	2.2167	3.1979	0.8388
DMU12	6.3465	0.7292	0.3308	1.6952	6.5839	2.5628	6.8627	1.4160	209.17	3.8388	2.0141	6.5670	12.494	16.809	5.0639	0.7511	0.7251	0.8384	3.1764	0.7429	13.059	3.4435	1.0000	6.2208	3.2941	1.9943	1.2279
DMU13	3.1937	1.1899	0.3742	5.6624	3.1736	2.1323	11.0864	1.7887	168.70	4.9678	4.2966	4.9693	7.5964	16.200	3.5793	1.8948	4.1498	0.5017	6.4345	0.8409	7.4161	5.1743	3.0000	1.0924	5.0085	4.4893	0.3419

Legend. I: Inputs. O: Outputs. M: Middle. a: Purchase. b: Human resources. c: Accounting and finance. d: Research and development. e: Delivery to customers. f: Advertise. g: Drain the stone. h: Stone cutting. i: Store.

TABLE 5. continued.

DMUs	I_a			O_a	I_b		O_b		I_c	O_c		I_d	O_d	I_e	O_e		I_f		O_f		I_g		Mo_g, I_h	I_h	Mo_h, I_i	O_i	
	1	2	3	1	1	2	1	2	1	1	2	1	1	1	1	2	1	2	1	2	1	2	1	1	1	1	2
DMU14	2.7045	1.1091	0.3953	3.1800	6.5428	2.5159	5.9200	1.9241	193.25	3.8318	2.0076	7.8290	12.430	6.6183	4.9316	1.1933	0.8308	0.8934	1.8203	0.8938	6.4467	4.1319	3.0000	8.5308	4.2042	3.5890	0.6657
DMU15	2.0871	0.6555	0.4024	6.1985	6.5309	3.1956	10.901	2.6558	162.80	0.5136	1.1447	7.7761	7.5055	6.4173	5.0107	0.6141	4.3634	0.7441	5.2315	0.9023	2.2915	3.1277	2.5000	1.2594	4.3045	4.9019	0.4216
DMU16	2.6578	0.5546	0.5678	3.8278	8.6352	3.3408	3.5460	2.0147	186.69	3.6580	1.8850	8.7095	12.471	16.588	5.5299	0.8327	0.8902	0.8680	2.9896	0.5257	2.7605	3.4090	2.7500	10.215	3.2996	3.0482	0.7652
DMU17	1.5272	0.5068	0.3679	1.0000	8.1757	3.1136	1.9206	1.7700	215.46	2.8048	2.2738	8.9936	2.6487	31.445	4.9021	0.3823	7.9662	0.3312	2.2584	0.5864	2.0343	3.5537	2.5000	13.333	3.7965	2.7693	0.8242
DMU18	1.6515	0.4980	0.3534	1.4074	7.8344	2.8187	1.1918	2.1025	211.64	4.5530	4.2553	8.9443	2.6914	41.703	4.9684	0.4001	7.9354	0.2776	2.8687	0.2443	0.7727	3.6372	3.5000	14.467	3.3219	0.7044	1.5336
DMU19	2.0067	1.4589	0.9078	9.1750	8.3881	3.5766	4.7153	2.0913	133.54	0.7004	0.9060	8.5668	2.6487	11.75	5.1883	0.4807	7.9068	0.3566	0.7557	0.6139	1.9101	3.3509	3.0000	9.3603	3.8077	2.9898	0.8691
DMU20	3.9879	0.4887	0.5429	4.7132	8.4147	3.1797	2.0453	1.6912	178.57	2.8151	2.4035	8.8546	2.6273	26.654	5.3921	0.4267	7.9816	0.3580	1.8525	0.5229	1.4421	3.7204	3.0000	12.449	4.3268	2.0577	1.1320
DMU21	2.9346	0.4881	0.5241	1.0877	7.9252	3.2771	2.5973	1.7658	214.48	2.5774	2.6079	8.3651	2.5631	26.937	5.2404	0.3209	8.0279	0.4383	3.0958	0.5553	0.9716	3.8669	2.5000	12.488	2.8473	0.8611	1.5022
DMU22	2.6051	1.1314	0.5018	10.8355	5.7613	3.1799	6.6391	2.1352	116.76	1.7448	3.2560	6.2274	7.5091	6.3876	4.6924	0.7242	4.3017	0.6370	2.4049	0.7098	2.6196	2.9982	2.5000	6.2349	3.2662	4.6874	0.2956
DMU23	1.5105	0.8139	0.5992	22.486	4.1095	3.3884	9.9802	0.9775	1.00	2.9285	6.6962	1.0000	4.9425	1.0000	4.8968	1.7847	6.1285	0.6032	4.0944	0.8995	11.257	2.1722	2.2500	5.4894	4.0370	6.9963	1.4057
DMU24	4.9879	1.4439	0.7505	7.1145	5.4412	2.2344	10.702	2.8874	154.75	3.2312	5.1273	2.1870	2.7342	11.382	5.0660	0.8676	7.7858	0.2749	4.7853	0.7450	2.5683	3.8283	3.0000	1.5448	4.2711	4.1693	0.8727
DMU25	0.5996	1.3565	0.4161	3.1702	7.8278	4.9528	7.9830	0.4920	193.94	5.3412	1.3391	7.4143	7.3772	6.5941	2.3982	1.1589	4.5747	0.8540	2.3646	0.6160	18.028	3.6971	1.0000	6.4843	2.1944	3.6625	0.6584
DMU26	1.4762	0.9965	0.4623	3.1348	5.6849	4.5748	8.7522	1.7299	194.54	2.5471	2.8330	7.1155	12.387	6.6142	5.0288	0.7057	0.8022	0.9723	1.8606	0.9876	3.1771	3.5774	3.0000	4.2601	3.7909	3.7845	0.6174

The inverse DEA models developed for the procurement department in the binary merger of industrial units (units m and n) according to the dual of the model for this department are given in equations (60)–(65):

$$\text{Min} \sum_{I}[\alpha_I(m) + \alpha_I(n)] \tag{60}$$

$$\sum_j \lambda_j x_{Ij} \leq \theta_{\text{Purchase}} * [\alpha_I(m) + \alpha_I(n)] \qquad \forall I \in \{1, 2, \ldots, i\} \tag{61}$$

$$\sum_j \lambda_j y^1_{L_1 j} \geq y^1_{L_1 m} + y^1_{L_1 n} \qquad \forall L_1 \in \{1, 2, \ldots, l_1\} \tag{62}$$

$$0 \leq \alpha_I(m) \leq x_{Im} \qquad \forall I \in \{1, 2, \ldots, i\} \tag{63}$$

$$0 \leq \alpha_I(n) \leq x_{In} \qquad \forall I \in \{1, 2, \ldots, i\} \tag{64}$$

$$\lambda_j \geq 0 \qquad \forall j \in \{1, 2, \ldots, N\}. \tag{65}$$

Table 6. Variables and parameters used in the NDEA model.

Parameters		Variables	
x_{ij}	Value of input i of the procurement department for unit j	s_i	x_{ij} criteria weight
$y^1_{l_1j}$	Value of output l_1 of the procurement department for unit j	$v^1_{l_1}$	$y^1_{l_1j}$ criteria weight
$y^2_{l_2j}$	Value of input l_2 of the human resources management department for unit j	$v^2_{l_2}$	$y^2_{l_2j}$ criteria weight
$y^3_{l_3j}$	Value of input l_3 of the accounting and finance department for unit j	$v^3_{l_3}$	$y^3_{l_3j}$ criteria weight
$y^4_{l_4j}$	Value of input l_4 of the research and development department for unit j	$v^4_{l_4}$	$y^4_{l_4j}$ criteria weight
$z^1_{f_1j}$	Value of output f_2 of the human resources management department for unit j	$w^1_{f_1}$	$z^1_{f_1j}$ criteria weight
$z^2_{f_2j}$	Value of output f_3 of the accounting and finance department for unit j	$w^2_{f_2}$	$z^2_{f_2j}$ criteria weight
$z^3_{f_3j}$	Value of output f_4 of the research and development department for unit j	$w^3_{f_3}$	$z^3_{f_3j}$ criteria weight
$z^4_{u_1j}$	Value of input u_1 of the rock offloading department for unit j	$w^4_{u_1}$	$z^4_{u_1j}$ criteria weight
$z^5_{u_2j}$	Value of output u_2 of the rock offloading department (input of the stone cutting department) for unit j	$w^5_{u_2}$	$z^5_{u_2j}$ criteria weight
$z^6_{u_3j}$	Value of input u_3 of the stone cutting department for unit j	$w^6_{u_3}$	$z^6_{u_3j}$ criteria weight
$z^7_{u_4j}$	Value of output u_4 of the stone cutting department (input of the storage and packaging department) for unit j	$w^7_{u_4}$	$z^7_{u_4j}$ criteria weight
$T^1_{u_5j}$	Value of output u_5 of the storage and packaging department for unit j	$H^1_{u_5}$	$T^1_{u_5j}$ criteria weight
$T^2_{R_2j}$	Value of input R_2 of the delivery department for unit j	$H^2_{R_2}$	$T^2_{R_2j}$ criteria weight
$T^3_{R_3j}$	Value of input R_3 of the marketing and advertising department for unit j	$H^3_{R_3}$	$T^3_{R_3j}$ criteria weight
$P^1_{s_1j}$	Value of output s_1 of the delivery department for unit j	$E^1_{s_1}$	$P^1_{s_1j}$ criteria weight
$P^2_{s_2j}$	Value of output s_2 of the marketing and advertising department for unit j	$E^2_{s_2}$	$P^2_{s_2j}$ criteria weight

Notes. All parameters and variables have been used in relations 32–55. Subscripts and superscripts are used only to name them.

The dual of the DEA models for the support department based on the CRS model are expressed in equations (66)–(73):

$$\text{Min } \theta_{\text{Support}} \tag{66}$$

$$\sum_j \lambda^1_j y^2_{L_2j} \le \theta_{\text{Support}} * y^2_{L_2k} \qquad \forall L_2 \in \{1,2,\dots,l_2\} \tag{67}$$

$$\sum_j \lambda^2_j y^3_{L_3j} \le \theta_{\text{Support}} * y^3_{L_3k} \qquad \forall L_3 \in \{1,2,\dots,l_3\} \tag{68}$$

$$\sum_j \lambda^3_j y^4_{L_4j} \le \theta_{\text{Support}} * y^4_{L_4k} \qquad \forall L_4 \in \{1,2,\dots,l_4\} \tag{69}$$

$$\sum_j \lambda^1_j z^1_{F_1j} \ge z^1_{F_1k} \qquad \forall F_1 \in \{1,2,\dots,f_1\} \tag{70}$$

$$\sum_j \lambda^2_j z^2_{F_2j} \ge z^2_{F_2k} \qquad \forall F_2 \in \{1,2,\dots,f_2\} \tag{71}$$

$$\sum_j \lambda^3_j z^3_{F_3j} \ge z^3_{F_3k} \qquad \forall F_3 \in \{1,2,\dots,f_3\} \tag{72}$$

$$\lambda^1_j, \lambda^2_j, \lambda^3_j \ge 0 \qquad \forall j \in \{1,2,\dots,N\}. \tag{73}$$

TABLE 7. Efficiency of each sub-network according to the NDEA model.

DMUs	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10	DMU11	DMU12	DMU13	DMU14	DMU15	DMU16	DMU17	DMU18	DMU19	DMU20	DMU21	DMU22	DMU23	DMU24	DMU25	DMU26
Procurement	1.000000	1.000000	0.176522	0.192135	0.278559	0.276936	0.318786	0.326759	0.097497	0.101197	0.114607	0.089381	0.24897	0.132339	0.351183	0.249824	0.071575	0.102865	0.258274	0.349109	0.080652	0.36902	1.000000	0.186719	0.199462	0.125023
Support	0.476755	1.000000	0.29827	0.460705	1.000000	1.000000	0.401688	0.642952	0.450823	1.000000	0.440955	0.427878	1.000000	0.576216	0.711361	0.455538	0.426595	0.54642	0.461843	0.397841	0.418554	0.604461	1.000000	1.000000	0.298088	0.423349
Manufacturing	0.571542	0.181729	1.000000	0.581313	1.000000	0.383712	0.471282	0.332283	0.226737	1.000000	0.266789	0.314248	0.406789	0.385136	1.000000	0.688768	0.782405	1.000000	0.89003	0.889498	0.961581	0.927101	0.732959	0.947305	0.228893	0.677459
Sale	0.780803	0.513912	0.388397	0.863028	0.802007	0.258819	0.306507	0.551223	0.894393	1.000000	0.932001	1.000000	1.000000	0.977285	0.821856	0.864414	0.550895	0.593501	0.517827	0.453214	0.516536	0.683766	1.000000	1.000000	0.491228	1.000000

The inverse DEA models formulated for the support department in the binary merger of industrial units (units m and n) according to the dual of the model for this department are as follows:

$$\text{Min}\left[\sum_{L_2}\alpha^2_{L_2}(m)+\alpha^2_{L_2}(n)+\sum_{L_3}\alpha^3_{L_3}(m)+\alpha^3_{L_3}(n)+\sum_{L_4}\alpha^4_{L_4}(m)+\alpha^4_{L_4}(n)\right] \tag{74}$$

$$\sum_j \lambda^1_j y^2_{L_2 j} \le \theta_{\text{Support}} * \left[\alpha^2_{L_2}(m)+\alpha^2_{L_2}(n)\right] \quad \forall L_2 \in \{1,2,\dots,l_2\} \tag{75}$$

$$\sum_j \lambda^2_j y^3_{L_3 j} \le \theta_{\text{Support}} * \left[\alpha^3_{L_3}(m)+\alpha^3_{L_3}(n)\right] \quad \forall L_3 \in \{1,2,\dots,l_3\} \tag{76}$$

$$\sum_j \lambda^3_j y^4_{L_4 j} \le \theta_{\text{Support}} * \left[\alpha^4_{L_4}(m)+\alpha^4_{L_4}(n)\right] \quad \forall L_4 \in \{1,2,\dots,l_4\} \tag{77}$$

$$\sum_j \lambda^1_j z^1_{F_1 j} \ge z^1_{F_1 m}+z^1_{F_1 n} \quad \forall F_1 \in \{1,2,\dots,f_1\} \tag{78}$$

$$\sum_j \lambda^2_j z^2_{F_2 j} \ge z^2_{F_2 m}+z^2_{F_2 n} \quad \forall F_2 \in \{1,2,\dots,f_2\} \tag{79}$$

$$\sum_j \lambda^3_j z^3_{F_3 j} \ge z^3_{F_3 m}+z^3_{F_3 n} \quad \forall F_3 \in \{1,2,\dots,f_3\} \tag{80}$$

$$0 \le \alpha^2_{L_2}(m) \le y^2_{L_2 m} \quad \forall L_2 \in \{1,2,\dots,l_2\} \tag{81}$$

$$0 \le \alpha^3_{L_3}(m) \le y^3_{L_3 m} \quad \forall L_3 \in \{1,2,\dots,l_3\} \tag{82}$$

$$0 \le \alpha^4_{L_4}(m) \le y^4_{L_4 m} \quad \forall L_4 \in \{1,2,\dots,l_4\} \tag{83}$$

$$0 \le \alpha^2_{L_2}(n) \le y^2_{L_2 n} \quad \forall L_2 \in \{1,2,\dots,l_2\} \tag{84}$$

$$0 \le \alpha^3_{L_3}(n) \le y^3_{L_3 n} \quad \forall L_3 \in \{1,2,\dots,l_3\} \tag{85}$$

$$0 \le \alpha^4_{L_4}(n) \le y^4_{L_4 n} \quad \forall L_4 \in \{1,2,\dots,l_4\} \tag{86}$$

$$\lambda^1_j,\lambda^2_j,\lambda^3_j \ge 0 \quad \forall j \in \{1,2,\dots,N\}. \tag{87}$$

The dual of the DEA models for the manufacturing department based on the CRS model are given in equations (88)–(94):

$$\text{Min } \theta_{\text{Produce}} \tag{88}$$

$$\sum_j \left(\lambda_j^4 + P_j\right) z_{U_1 j}^4 \leq \theta_{\text{Produce}} * Z_{U_1 k}^4 \qquad \forall U_1 \in \{1, 2, \ldots, u_1\} \tag{89}$$

$$\sum_j \left(\lambda_j^6 + P_j\right) z_{U_3 j}^6 \leq \theta_{\text{Produce}} * Z_{U_3 k}^6 \qquad \forall U_3 \in \{1, 2, \ldots, u_3\} \tag{90}$$

$$\sum_j \lambda_j^5 z_{U_2 j}^5 \geq \sum_j \lambda_j^6 z_{U_2 j}^5 \qquad \forall U_2 \in \{1, 2, \ldots, u_2\} \tag{91}$$

$$\sum_j \lambda_j^7 z_{U_4 j}^7 \geq \sum_j \lambda_j^8 z_{U_4 j}^7 \qquad \forall U_4 \in \{1, 2, \ldots, u_4\} \tag{92}$$

$$\sum_j \left(\lambda_j^8 + P\right) T_{U_5 j}^1 \geq T_{U_5 k}^1 \qquad \forall U_5 \in \{1, 2, \ldots, u_5\} \tag{93}$$

$$\lambda_j^4, \lambda_j^5, \lambda_j^6, \lambda_j^7, \lambda_j^8, P_j \geq 0 \qquad \forall j \in \{1, 2, \ldots, N\}. \tag{94}$$

The inverse DEA models developed for the manufacturing department in the binary merger of industrial units (units m and n) according to the dual of the model for this department are:

$$\text{Min } \left[\sum_{U_1} \rho_{U_1}^4(m) + \rho_{U_1}^4(n) + \sum_{U_3} \rho_{U_3}^6(m) + \rho_{U_3}^6(n)\right] \tag{95}$$

$$\sum_j \left(\lambda_j^4 + P_j\right) z_{U_1 j}^4 \leq \theta_{\text{Produce}} * \left[\rho_{U_1}^4(m) + \rho_{U_1}^4(n)\right] \qquad \forall U_1 \in \{1, 2, \ldots, u_1\} \tag{96}$$

$$\sum_j \left(\lambda_j^6 + P_j\right) z_{U_3 j}^6 \leq \theta_{\text{Produce}} * \left[\rho_{U_3}^6(m) + \rho_{U_3}^6(n)\right] \qquad \forall U_3 \in \{1, 2, \ldots, u_3\} \tag{97}$$

$$\sum_j \lambda_j^5 z_{U_2 j}^5 \geq \sum_j \lambda_j^6 z_{U_2 j}^5 \qquad \forall U_2 \in \{1, 2, \ldots, u_2\} \tag{98}$$

$$\sum_j \lambda_j^7 z_{U_4 j}^7 \geq \sum_j \lambda_j^8 z_{U_4 j}^7 \qquad \forall U_4 \in \{1, 2, \ldots, u_4\} \tag{99}$$

$$\sum_j \left(\lambda_j^8 + P_j\right) T_{U_5 j}^1 \geq T_{U_5 m}^1 + T_{U_5 n}^1 \qquad \forall U_5 \in \{1, 2, \ldots, u_5\} \tag{100}$$

$$0 \leq \rho_{U_1}^4(m) \leq z_{U_1 m}^4 \qquad \forall U_1 \in \{1, 2, \ldots, u_1\} \tag{101}$$

$$0 \leq \rho_{U_3}^6(m) \leq z_{U_3 m}^6 \qquad \forall U_3 \in \{1, 2, \ldots, u_3\} \tag{102}$$

$$0 \leq \rho_{U_1}^4(n) \leq z_{U_1 n}^4 \qquad \forall U_1 \in \{1, 2, \ldots, u_1\} \tag{103}$$

$$0 \leq \rho_{U_3}^6(n) \leq z_{U_3 n}^6 \qquad \forall U_3 \in \{1, 2, \ldots, u_3\} \tag{104}$$

$$\lambda_j^4, \lambda_j^5, \lambda_j^6, \lambda_j^7, \lambda_j^8 \geq 0 \qquad \forall j \in \{1, 2, \ldots, N\}. \tag{105}$$

The dual of the DEA models for the sales department based on the CRS model are given below:

$$\text{Min } \theta_{\text{Sale}} \tag{106}$$

$$\sum_j \lambda_j^9 T_{R_2 j}^2 \leq \theta_{\text{Sale}} * T_{R_2 k}^2 \qquad \forall R_2 \in \{1, 2, \ldots, r_2\} \tag{107}$$

$$\sum_j \lambda_j^{10} T_{R_3 j}^3 \leq \theta_{\text{Sale}} * T_{R_3 k}^3 \qquad \forall R_3 \in \{1, 2, \ldots, r_3\} \tag{108}$$

$$\sum_j \lambda_j^9 P^1_{S_1 j} \geq p^1_{S_1 k} \qquad \forall S_1 \in \{1, 2, \ldots, s_1\} \tag{109}$$

$$\sum_j \lambda_j^{10} P^2_{S_2 j} \geq p^2_{S_2 k} \qquad \forall S_2 \in \{1, 2, \ldots, s_2\} \tag{110}$$

$$\lambda_j^9, \lambda_j^{10} \geq 0 \qquad \forall j \in \{1, 2, \ldots, N\}. \tag{111}$$

The inverse DEA models obtained for the sales department in the binary merger of industrial units (units m and n) according to the dual of the model for this department are provided in equations (112)–(121):

$$\text{Min } \left[\sum_{R_2} \beta^2_{R_2}(m) + \beta^2_{R_2}(n) + \sum_{R_3} \beta^3_{R_3}(m) + \beta^3_{R_3}(n)\right] \tag{112}$$

$$\sum_j \lambda_j^9 T^2_{R_2 j} \leq \theta_{\text{Sale}} * \left[\beta^2_{R_2}(m) + \beta^2_{R_2}(n)\right] \qquad \forall R_2 \in \{1, 2, \ldots, r_2\} \tag{113}$$

$$\sum_j \lambda_j^{10} T^3_{R_3 j} \leq \theta_{\text{Sale}} * \left[\beta^3_{R_3}(m) + \beta^3_{R_3}(n)\right] \qquad \forall R_3 \in \{1, 2, \ldots, r_3\} \tag{114}$$

$$\sum_j \lambda_j^9 P^1_{S_1 j} \geq p^1_{S_1 m} + p^1_{S_1 n} \qquad \forall S_1 \in \{1, 2, \ldots, s_1\} \tag{115}$$

$$\sum_j \lambda_j^{10} P^2_{S_2 j} \geq p^2_{S_2 m} + p^2_{S_2 n} \qquad \forall S_2 \in \{1, 2, \ldots, s_2\} \tag{116}$$

$$0 \leq \beta^2_{R_2}(m) \leq T^2_{R_2 m} \qquad \forall R_2 \in \{1, 2, \ldots, r_2\} \tag{117}$$

$$0 \leq \beta^3_{R_3}(m) \leq T^3_{R_3 m} \qquad \forall R_3 \in \{1, 2, \ldots, r_3\} \tag{118}$$

$$0 \leq \beta^2_{R_2}(n) \leq T^2_{R_2 n} \qquad \forall R_2 \in \{1, 2, \ldots, r_2\} \tag{119}$$

$$0 \leq \beta^3_{R_3}(n) \leq T^3_{R_3 n} \qquad \forall R_3 \in \{1, 2, \ldots, r_3\} \tag{120}$$

$$\lambda_j^9, \lambda_j^{10} \geq 0 \qquad \forall j \in \{1, 2, \ldots, N\}. \tag{121}$$

After coding the inverse DEA models in GAMS, the residuals of each input for each unit to be merged was obtained, and the percentage change in each criterion was determined accordingly.

As previously explained, the data used in the DEA model were first subjected to a transformation to make them positive and then weighted based on the percentage of variance they preserve. Therefore, in the model, $\text{PC}(w)_i$ was used instead of PC_i, resulting in the following formulation:

$$\text{PC}(w)_i = (\text{PC}_i + \{-\text{Min} + 1\}) * \text{Weight}_i \tag{122}$$

$$\Delta\text{PC}(w)_i = \text{Weight}_i * \Delta\text{PC}_i. \tag{123}$$

The results are presented in Tables 8–11.

Because of the use of PCA to reduce the dimensionality of the data to be fed into DEA, the obtained changes are related to the selected principal components and not the original problem data. Therefore, following the principles of PCA, the new inputs of the units to be merged were determined by the use of eigenvectors, which were obtained from equation (124) (where a_{ij} denotes eigenvectors)

$$\text{PC}_i = a_{i1}x_1 + a_{i2}x_2 + \ldots + a_{ij}x_j. \tag{124}$$

The eigenvectors obtained by implementing PCA in MATLAB are given in Tables 12–19.

Knowing the percentage change in each principal component after a merger of an industrial unit, one can estimate the amount of change to be expected in each of the main criteria to help the managers of the industry make more informed decisions about the modification of their inputs in order to achieve a target performance level. For example, to estimate the amount of change in each of the three inputs of the marketing and advertisement department in the sales network, considering that some criteria are not applicable, one should use

Table 8. Percentage change in each criterion in the raw material and consumable procurement department.

Target Eff.	1		0.125023		0.351183		0.36902		0.258274		1	
DMUs	1	5	10	26	12	15	13	22	14	19	23	24
PC1(w) Remaining	0.591	0	1.512	0.695	0.617	0	2.384	0	1.685	0	0.813	0
PC2(w) Remaining	0.66	0.569	0.74	0.996	0.729	0.556	1.19	0.724	1.109	1.42	0.814	0.878
PC3(w) Remaining	0.281	0.375	0.388	0.402	0.331	0.355	0.374	0.502	0.395	0.908	0.599	0.304
ΔPC1(w)	−43.01	−100	0	−52.92	−90.28	−100	−25.36	−100	−37.71	−100	−46.16	−100
ΔPC2(w)	0	−1.04348	0	0	0	−15.1145	0	−36.0064	0	−2.67306	0	−39.1967
ΔPC3(w)	0	−70.6768	0	−13.0436	0	−11.7777	0	0	0	0	0	−59.4924
ΔPC1	−73.63	−171.21	0	−90.6	−154.56	−171.21	−43.42	−171.21	−64.56	−171.21	−79.03	−171.21
ΔPC2	0	−4.32104	0	0	0	−62.589	0	−149.102	0	−11.0691	0	−162.313
ΔPC3	0	−405.194	0	−74.7796	0	−67.5222	0	0	0	0	0	−341.074

Table 9. Percentage change in each criterion in the manufacturing department.

	Target Eff.	1		1		1		0.927101		0.89003		0.947305	
	DMUs	1	5	10	26	12	15	13	22	14	19	23	24
Offload	PC1(w) Remaining	0.37	0	0.135	0	0.198	0	0.16	0	0.208	0	0.296	0
	PC2(w) Remaining	1.742	0	0.545	0.093	0.931	0	0.753	0	0.977	0	1.392	0
Cutting	PC1(w) Remaining	1.003	0	0.367	0	0.536	0	0.434	0	0.563	0	0.802	0
Offload	ΔPC1(w)	−96.31	−100	−98.88	−100	−98.48	−100	−97.84	−100	−96.77	−100	−97.37	−100
	ΔPC2(w)	−22.3079	−100	0	−97.4003	−72.9632	−100	−85.4472	−100	−76.3545	−100	−35.9187	−100
Cutting	ΔPC1(w)	−87.3468	−100	−96.6694	−100	−91.3838	−100	−60.2726	−100	−93.4004	−100	−85.39	−100
Offload	ΔPC1	−124.65	−129.424	−127.97	−129.424	−127.46	−129.424	−126.63	−129.424	−125.25	−129.424	−126.02	−129.42
	ΔPC2	−98.124	−439.862	0	−428.427	−320.938	−439.862	−375.85	−439.862	−335.855	−439.862	−157.993	−439.86
Cutting	ΔPC1	−87.3468	−100	−96.6694	−100	−91.3838	−100	−60.2726	−100	−93.4004	−100	−85.39	−100

Table 10. Percentage change in each criterion in the support department.

	Target Eff.	1		1		0.711361		1		0.576216		1	
	DMUs	1	5	10	26	12	15	13	22	14	19	23	24
Human	PC1(w) Remaining	3.764	0	1.598	1.641	6.584	1.765	3.174	0.576	6.543	5.186	4.109	1.154
Res.	PC2(w) Remaining	3.933	2.085	0.427	2.555	2.563	3.196	2.132	2.928	2.516	3.577	3.388	1.409
Fin.	PC1(w) Remaining	3.553	0	2.314	0	2.089	0	2.292	0	2.686	0	1	1.103
R&D	PC1(w) Remaining	2.428	0	3.051	0	5.688	0	3.056	0	5.295	0	1	0.553
Human	ΔPC1(w)	−48.78	−100	0	−71.13	0	−72.97	0	−90	0	−38.17	0	−78.79
Res.	ΔPC2(w)	0	−35.984	0	−44.1503	0	0	0	−7.92453	0	0	0	−36.9411
Fin.	ΔPC1(w)	−93.5471	−100	−98.8798	−100	−99.0013	−100	−98.6414	−100	−98.6101	−100	0	−99.2872
R&D	ΔPC1(w)	−64.4456	−100	−56.7641	−100	−13.3851	−100	−38.5025	−100	−32.3665	−100	0	−74.7137
Human	ΔPC1	−85.08	−174.4	0	−124.05	0	−127.27	0	−156.96	0	−66.57	0	−137.41
Res.	ΔPC2	0	−84.3522	0	−103.495	0	0	0	−18.5763	0	0	0	−86.5956
Fin.	ΔPC1	−93.5471	−100	−98.8798	−100	−99.0013	−100	−98.6414	−100	−98.6101	−100	0	−99.2872
R&D	ΔPC1	−64.4456	−100	−56.7641	−100	−13.3851	−100	−38.5025	−100	−32.3665	−100	0	−74.7137

Table 11. Percentage change in each criterion in the sales department.

	Target Eff.	0.802007		1		1		1		0.977285		1	
	DMUs	1	5	10	26	12	15	13	22	14	19	23	24
Delivery	PC1(w) Remaining	1.179	1.082	2.231	0	2.057	0	1.689	0	2.115	0	1	1.035
AD.	PC1(w) Remaining	6.144	4.047	7.726	0.802	0.725	2.784	4.15	1.289	0.831	2.726	6.129	2.286
	PC2(w) Remaining	0.63	1.223	0.3	0.972	0.838	0.744	0.502	0.637	0.893	0.357	0.603	0.275
Delivery	ΔPC1(w)	0	−81.03	−86.29	−100	−87.76	−100	−89.57	−100	−68.04	−100	0	−90.91
AD.	ΔPC1(w)	0	−9.89375	0	0	0	−36.1964	0	−70.0352	0	−65.5234	0	−70.6387
	ΔPC2(w)	0	0	0	0	0	0	0	0	0	0	0	0
Delivery	ΔPC1	0	−81.03	−86.29	−100	−87.76	−100	−89.57	−100	−68.04	−100	0	−90.91
AD.	ΔPC1	0	−13.6439	0	0	0	−49.9165	0	−96.5816	0	−90.3597	0	−97.4139
	ΔPC2	0	0	0	0	0	0	0	0	0	0	0	0

TABLE 12. Eigenvectors related to principal components-inputs of the procurement department.

	Cr.1	Cr.2	Cr.3	Cr.4	Cr.5
PC_1	0.102349	0.161575	0.980626	−0.02107	−0.03669
PC_2	−0.02372	−0.02456	0.037028	−0.25949	0.964432
PC_3	−0.08321	−0.04154	0.045693	0.961668	0.253885

TABLE 13. Eigenvectors related to principal components-inputs of the human resources management department.

	Cr.1	Cr.2	Cr.3	Cr.4	Cr.5	Cr.6
PC_1	0.419782	0.131431	0.4468	0.772967	−0.07572	−0.06056
PC_2	−0.6072	−0.1189	−0.47535	0.622578	0.022423	−0.0557

TABLE 14. Eigenvectors related to principal components-inputs the accounting and finance department.

	Cr.1	Cr.2	Cr.3	Cr.4	Cr.5
PC_1	−0.09911	−0.30822	−0.13024	0.05774	0.935352

TABLE 15. Eigenvectors related to principal components-inputs of the research and development department.

	Cr.1	Cr.2	Cr.3	Cr.4
PC_1	0.046713	0.94709	0.193149	0.252057
PC_2	0.523907	−0.27844	0.683321	0.425516

TABLE 16. Eigenvectors related to principal components-inputs of the stone offloading department.

	Cr.1	Cr.2	Cr.3	Cr.4	Cr.5	Cr.6	Cr.7	Cr.8	Cr.9	Cr.10	Cr.11	Cr.12
PC_1	−0.03393	0.060067	−0.09722	0.021038	0.068616	−0.00301	0.142442	0.483835	0.388226	0.756561	−0.05598	0.002181
PC_2	0.031795	−0.1987	0.001326	0.021293	0.043373	0.014071	−0.03131	0.771952	0.187081	−0.56551	0.07264	0.001769

TABLE 17. Eigenvectors related to principal components-inputs of the stone cutting department.

	Cr.1	Cr.2	Cr.3	Cr.4	Cr.5
PC_1	0.11463	−0.01025	0.038341	0.992532	0.012863

equations (125) and (126):

$$\mathrm{PC}_1 = 0.212655X_1 - 0.97369X_2 - 0.081884X_3 \tag{125}$$

$$\mathrm{PC}_2 = 0.974383X_1 + 0.0.20504X_2 + 0.0924X_3. \tag{126}$$

Table 18. Eigenvectors related to principal components-inputs of the delivery department.

	Cr.1	Cr.2	Cr.3	Cr.4	Cr.5
PC_1	0.111286	0.040302	0.068309	0.990476	0.016795

Table 19. Eigenvectors related to principal components-inputs of the advertisement department.

	Cr.1	Cr.2	Cr.3
PC_1	0.212655	0.97369	0.081884
PC_2	0.974383	−0.20504	−0.0924

Table 20. New input values of industrial units after the merger.

Sections	DMU	1	5	10	26	12	15	13	22	14	19	23	24
Procurement	X_1	2	1	0.7	5.938	3	0.5	0.4	0.3	1	0.786	1	8.269
	X_2	1	1.760	4	0.199	0.315	1	0.471	1	1.5	0	0.788	2
	X_3	1	0	1.5	0	0	1.916	0	0.333	0.948	0	0	1.954
	X_4	0.5	2.299	1	0	0.792	0.128	0.356	2.023	0.606	3.31	2.063	0
	X_5	200	295.004	250	349.924	200.419	198.77	400.221	396.787	380.146	599.131	300.088	548.836
Human Res.	X_1	8	8	4	1	2	4	4	6	2	4	4	4
	X_2	20	20	13	9	8	7	15	8	10	6	20	8
	X_3	77.147	69.565	0	4.09	0	122.464	2	127.727	0	61.568	4	27.566
	X_4	65.032	197.106	15	165.702	5	97.927	10	133.759	5	50.186	7	168.326
	X_5	1	1	2	3	2	3	1	2	3	2	3	2
	X_6	4	4	3	3	4	3	1	2	2	3	3	4
Financial	X_1	90	75	70	70	60	70	70	75	70	70	90	80
	X_2	900	450	480	800	500	700	700	800	480	500	1200	800
	X_3	500	250	220	400	300	350	300	400	350	600	350	600
	X_4	80	90	70	70	70	90	80	85	85	80	80	70
	X_5	15 100.013	3607.529	955.714	936.912	605.844	4606.912	4105.459	9106.912	2105.426	8106.912	20 000	4806.15
R&D	X_1	4	4	5	3	5	4	1	2	5	5	4	2
	X_2	73.565	112.087	63.755	110.587	18.469	110.106	47.282	111.37	37.179	107.812	11	88.609
	X_3	2	5	5	2	5	2	3	3	5	5	4	2
	X_4	2	4	5	3	5	2	5	3	5	5	3	5
Offload	X_1	10	12	8	12	8	12	8	10	10	12	20	12
	X_2	30	30	30	30	30	30	30	30	30	30	30	30
	X_3	11 433.749	14 855.887	10 629.817	20 084.664	14 450.732	22 303.964	18 146.739	22 305.191	15 703.581	21 327.674	20 223.143	21 284.838
	X_4	30	35	25	30	35	30	30	30	30	35	25	35
	X_5	0	0	0	0	0	0	0	0	0	0	0	0
	X_6	4	3	3	2	2	1	4	3	2	2	4	2
	X_7	100	100	100	100	100	100	100	100	100	100	100	100
	X_8	0	0	0	0	0	0	0	0	0	0	0	0
	X_9	0	0	0	0	0	0	0	0	0	0	0	0
	X_{10}	310.151	911.892	195.329	805.251	704.499	838.159	699.469	838.589	653.635	827.71	420.09	824.154
	X_{11}	0	0	0	0	0	5	5	0	5	0	0	5
	X_{12}	3	4	3	4	4	4	4	4	3	4	4	4
Cutting	X_1	2	0	12	24	24	24	12	24	48	36	48	48
	X_2	1	3	1	1	4	2	1	2	1	2	1	2
	X_3	3	3	3	2	2	2	3	2	1	2	3	2
	X_4	96.004	115.752	102.397	112.752	102.071	115.752	75.726	110.752	102.103	107.752	97.032	115.752
	X_5	3	8	3	4	4.5	3	2.9	4	3	4.8	4	3.8
Delivery	X_1	0	0	0	0	0	0	0	0	0	0	0	0
	X_2	25	5.955	8.136	8.333	0	13.148	2.409	1.81	4.37	6.912	40	7.014
	X_3	15	100	100	100	100	100	100	100	100	100	50	100
	X_4	5	5	5	5	5	5	5	5	5	5	5	5
	X_5	2.5	10	10	10	5.898	15	5	10	10	15	2.5	5
AD.	X_1	15	27.255	7	12	7	3.539	4	0.555	10	0.521	14	0.568
	X_2	2.5	18.497	0	10	10	53.812	5	48.985	10	49.348	2.5	29.663
	X_3	3	4	4	3	3	5	4	5	2	3	3	5

Having ΔPC_1 and ΔPC_2, equations (127) and (128) can be used to determine ΔX_i.

$$\Delta PC_1 = 0.212655\Delta X_1 - 0.97369\Delta X_2 - 0.081884\Delta X_3 \tag{127}$$

$$\Delta PC_2 = 0.974383\Delta X_1 + 0.0.20504\Delta X_2 + 0.0924\Delta X_3. \tag{128}$$

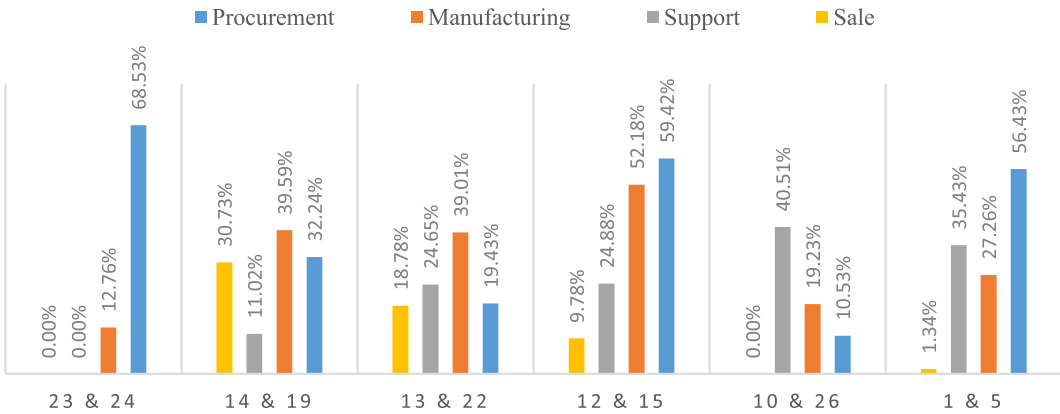

FIGURE 4. Percentage increase in the efficiency of units after the merger.

In this paper, these changes are limited in order to minimize the change in the original problem data. This has been achieved by applying some constraints to the extent of change possible in each input. For example, for percentage-type criteria, the change interval should be such that input values remain between zero and 100. As another example, for qualitative data that have been quantified based on the Likert scale, inputs should be discrete values ranging from 1 to 5. These constraints along with the objective function were analyzed in a mathematical model developed in GAMS. The results are presented in Table 20. As indicated in the previous sections, 6 optimal combinations were obtained for integration. For example, according to the objective functions and constraints of the integration model, it was concluded that units 1 and 5 meet the integration conditions. Since in this article it is assumed that after the merger of two units, none of them will be removed, the remaining amount of each of the criteria should be obtained for each of the units 1 and 5. The data of this the table shows how much of each of its inputs should be retained after unit 1 is merged with unit 5 to achieve the selected target performance in cooperation with the other unit.

The application of the proposed model to real data and the examination of its effect on the efficiency of industrial units showed an efficiency improvement compared to the average efficiency of units before the merger; a difference that is illustrated in Figure 4.

5. Conclusions and suggestions

Recognizing the importance of attention to SMEs operating in different industries and their growth potentials, many governments have adopted support policies and incentives to help them achieve higher efficiency levels. The efforts to enhance the efficiency of SMEs in an industry cannot succeed without a sound knowledge of the supply chain network of that industry and the criteria that influence its efficiency. Since the use of a large number of criteria tends to result in more logical conclusions, this study attempted to consider as many criteria as possible. However, since using large volumes of data greatly undermines the ability of DEA models to differentiate between DMUs, the PCA method was used to reduce the dimensionality of the problem while preserving the information contained in the data as much as possible. After developing NDEA models for evaluating the efficiency of DMUs, the paper explored the subject of the merger of industrial units with attention to efficiency criteria and certain constraints such as the location of units so that they can benefit from each other's competitive advantages, experiences, and facilities, modernize their equipment with joint capital, gain more resistant against unforeseen changes, and try to improve the quality of their products. After determining the optimal combination of units for the merger by the mathematical model, changes in the inputs of each sub-network of the merged units were estimated and then used to predict the increase in the efficiency level of the merged units relative to their pre-merger conditions. One of the innovations of this paper was the implementation of the model on the real data pertaining to the travertine stone industry and the identification of the most important criteria affecting this industry, which is welcomed by many units operating in this field of stone cutting and processing. As mentioned in the previous section, The application of the proposed model to real data and the examination of its effect on the efficiency of industrial units showed an efficiency improvement compared to the average efficiency of units before the merger; a difference that is illustrated in Figure 4.

Based on the results of this paper, the following recommendations can be made for future research. Firstly, probabilistic models and fuzzy data can be used to develop models with more realistic assumptions. Also, the model of this paper can be implemented in different organizations and industries and can also be utilized in the clustering of companies in different industries. In addition to including a larger number of units located in industrial towns across the country in the model, future works can modify the model so that the quality of travertine is also considered in the merger, especially since many of the parameters used in this article can be affected by the quality of stone blocks.

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