

An integrated model of network Data Envelopment Analysis and principal component analysis approach to calculate the efficiency of industrial units (Case study: Stone Industry)

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Abstract

Evaluating the efficiency of industrial units has long been an important issue to find the position of each unit in comparison with others. In this paper, a model for evaluating the efficiency using data envelopment analysis approach is explained in such a way that due to the breadth of input and output criteria, using the principal component analysis approach, data dimensions can also be reduced and the power to distinguish between efficient and inefficient units be increased. Due to lack of attention to the internal structure and also not considering the effective criteria in each department, it is tried to determine the most important criteria involved in each part in the purchasing, production, support and sales sectors. To calculate the efficiency, all the components have been examined as a model of network data envelopment analysis to take into account the effect of all departments and criteria in industrial units' efficiency. In this network, by considering the criteria involved in each of the sub-networks, all effective factors were identified. These criteria are selected based on the SCOR model and the balanced scorecard and also include sustainability criteria. To implement the model, 26 stone factories have been considered. The supply chain network was determined and dimensions of the data were reduced by implementing the principal component analysis approach. Then, by modeling the data envelopment analysis in each of the subnets in GAMS software, the efficiency was calculated. The results show an acceptable difference among industrial units to evaluate those units.

Keywords: Supply chain, Data Envelopment Analysis, Principal Component Analysis, SCOR, balanced scorecard, sustainability.

1-Introduction

Evaluating the performance and calculating the efficiency of companies and industrial units has long been an important issue to find the position of each decision-making unit compared to other units so that they can eliminate their weaknesses. To achieve these goals, data envelopment analysis approach has been used. In the DEA method, when the number of input and output variables is large, the result obtained from the model will be the efficiency of a large number of decision units. This greatly reduces the power of

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distinguishing between efficient and inefficient units. Therefore, achieving an appropriate number of variables for an acceptable result in data envelopment analysis is of great importance.

In this paper, to solve this problem, a combination of two approaches on network data envelopment analysis and principal component analysis is used. The most important goal of combining these two approaches is to reduce the size of the data in order to increase the recognition power of the model to find efficient and inefficient units.

Using a combination of DEA and PCA approaches is a way to reduce the size of the data set through the linear combination of variables and their variance structure (Adler & Golany, 2001). In the principal component analysis method, each principal component (obtained from the linear weight composition of the principal variables and arranged in order of variance percentage) contains the maximum variance while not related to the previous main components (Annapoorni & Prakash, 2017). Basically, PCA is designed to reduce the number of variables to a small number of indicators, called principal components and are a linear combination of the main variables. Therefore, the scores of the main component can be used instead of the main input and output variables (Adler & Golany, 2001).

In this paper, for the first time, the travertine stone industry is examined using real data. In the second part of this article, a review of the literature on the subject is reviewed. The third section examines the theoretical foundations and the proposed model. The fourth and fifth sections also present the case study and the conclusion, respectively.

2-Literature review

2-1-Review of Network Data Envelopment Analysis (NDEA)

Data envelopment analysis is a method of measuring the productivity of a decision unit regardless of its internal structure. Ignoring the internal processes of a decision unit will lead to the loss of some data and inaccurate results. Network data envelopment analysis method has been considered in various articles in the field of supply chain evaluation and in recent years, many researches have conducted it. The first article to introduce and develop this idea was Charnes', who found that military service consisted of two processes: The first process is informing through advertising and the second process is creating a contract (Charnes, A., Cooper, W.W., Golany, B., Halek, R., Klopp, G., Schmitz, E. and Thomas, 1986). As Seifard and Zhou conducted in 1999 on a two-stage baseline model, Soteriou & Zenios examined the quality of operations and marketing of 144 commercial bank branches in Cyprus (Soteriou & Zenios, 1999).

Löthgren and Tambour "considered customer satisfaction in a study of the performance of 31 pharmacies in Sweden. The system consists of two processes of production and consumption, and the allocation of a specific company as input for each process is allowed (Löthgren & Tambour, 1999). Golany et al. examined three models for measuring the efficiency of a two-stage system with common inputs. The system allows any process to receive inputs from other sources in exchange for the delivery of appropriate intermediate or final products (Golany et al., 2006).

Kao and Hwang described insurance companies (other than life) as having two-stage processes in which operating costs and insurance were the first input and premiums were the output of the first phase. In the second stage, the output was commitment and return on investment (Kao & Hwang, 2008).

Xu used this method to evaluate the performance of the furniture industry supply chain in China. He used cost indicators (direct, operating and shipping costs), order delay time, number of employees, flexibility, finance, order execution rate and delivery time percentage (Xu et al., 2009).

Kao discussed the series structures. He also described general structures. Then, by introducing artificial processes, he turned the system into a series system in which each process includes several parallel processes. He continued his previous research by examining systems with structures in more than two stages (Kao, 2009). Cook and his colleagues developed a model based on the nature of decision-making units, which are networked and multi-stage (Cook et al., 2010).

Lozano examined a network system with a general structure and analyzed its costs and returns on a scale (Lozano, 2011). Lozano and colleagues divided the operation of an airport into two parts and studied the performance of 39 airports in Spain: the movement of the aircraft and its landing, taking into account the undesirable outputs obtained from the first process (Lozano et al., 2013).

Wang et al. used the network DEA approach to measure the efficiencies of Chinese commercial banks using two-stage analyses. In the first stage they evaluated the deposit-producing process and in the second

stage the profit-earning process. In the first stage, the input variables were fixed assets and the number of employees while the output variable was the amount of bank deposits. The amount of bank deposits was the intermediate input/output variable. The second stage outputs were non-interest incomes, interest incomes, and one undesirable output non-performing loan. (Wang et al., 2014)

Another article developed in 2021 adopts an extended two-stage network DEA approach to measure the operating efficiency of 52 universities in China using a data set in 2014.(Chen et al., 2021)

In 2022, network DEA was used to evaluate bank performance. In this study, a two-stage network was developed. The research was divided into two parts. The first part determined the efficiency of the intermediate function of banks. The second part approached the regression analysis in which we determined the influence of the bank size, type of bank, and mergers and acquisitions activity on the defined efficiency. In the first stage the output-oriented DEA model was applied using deposits, labor costs, and capital as input variables; on the other side, loans and investments were used as output variables.(Milenković et al., 2022)

Domestic researchers have also done a lot of research in this area. In a study conducted by Olfat et al., using network data envelopment analysis, they evaluated the entire supply chain based on financial, knowledge, participation and accountability indicators in pharmaceutical companies listed in Tehran Stock Exchange (Azbari et al., 2014).

Mir Hedayatian et al. proposed a new method based on network data envelopment analysis to evaluate the green supply chain despite dual factors, adverse outputs, and fuzzy data, and implemented their model in 10 Iranian beverage companies (Mirhedayatian et al., 2014). A paper published by Matin et al. in 2015 presented a new development in network data envelopment analysis despite of general structures that require no specific assumptions about the interrelationships of processes. In this paper, a method for improving inefficient units and making efficient units was also presented (Matin & Azizi, 2015).

2-2-Review of Principal Component Analysis (PCA) and DEA

Adler and Golany have used a combination of data envelopment analysis and principal component analysis to reduce dimensions when the number of inputs and outputs are much larger than the decision units (Adler & Golany, 2007). Principal components can be used to replace all inputs or outputs simultaneously or to replace specific groups of variables (Adler & Yazhemsky, 2010).

PCA is a multivariate technique used to analyze the relationships of variables and explain the variables based on their components (Jr et al., 2014). To avoid subjectivity in selecting the index, PCA (principal component analysis) was used to reduce the dimensions and obtain comprehensive and objective indicators, followed by performance evaluation and analysis (Pöldaru & Roots, 2014).

Jothimani et al. used financial ratios as input and output performance evaluation parameters by PCA-DEA in an Indian stock market study (Jothimani et al., 2017). Jakaitienė et al. Proposed the PCA-DEA evaluation method to evaluate the performance of the European education system (JAKAITIENE et al., 2018). In a paper developed in 2020, a combination of data envelopment analysis and principal component analysis was used to evaluate the performance of 100 companies based on their 2015 financial statements (Bayaraa et al., 2020).

In 2020, an article was developed with the aim of considering carbon emission factors in the evaluation of logistics performance. In this paper, two approaches on principal component analysis and DEA covering approach were used (Deng et al., 2020). In another paper, the financial performance in 46 financial institutions was calculated using a combination of two approaches: data envelopment analysis and principal component analysis (Chauhan, 2021). In 2021, the confidence area method was used in data envelopment analysis and combined with the PCA approach to evaluate the energy security performance of 125 countries for 21 consecutive years (Wu et al., 2021).

Andrews combines principal component analysis and data envelopment intertemporal analysis with the double bootstrap approach to estimate the technical efficiency of health care providers along with the trend of efficiency performances. Empirically, this study on New Zealand DHBS shows that PCA and DEA intertemporal analysis offers a simple and insightful way to assess efficiency where there is a limited number of DMUs relative to input/output types.(Andrews, 2022)

In the paper in 2022, the efficiency analysis of transport companies is performed using an integrated PCA–DEA model and multi-criteria decision-making methods. The research was conducted on a sample of two transport companies from Bosnia and Herzegovina. In this paper, the PCA model is also used to

create new principal components that are linear combinations of initial variables, thus increasing the power of the DEA model.(Stević et al., 2022)

Internal researchers have also explored and expanded the principal component analysis approach. Azadeh examined an integrated framework for evaluating and ranking manufacturing systems - based on management and organizational performance indicators - for Iran's telecommunications sector (Azadeh et al., 2007). The first major component had the highest variance in the sample data. The second component had the second level of variance and so the principal components are calculated (Ahmadvand et al., 2011). Ahmadvand used PCA to implement a DEA model to reduce the dependence of variables (Ahmadvand et al., 2011). Tavakoli and Shirouyehzad used PCA-DEA to evaluate the performance of the steel company based on its human capital management (Tavakoli & Shirouyehzad, 2013).

Most reviewed articles emphasized on the combination of principal component analysis approach with data envelopment analysis approach. Also, network DEA has been less considered by researchers. In most of the studied articles, simple data envelopment analysis is combined with the principal component analysis approach and the relevant issue is investigated. In several articles that use network data envelopment analysis, only a two-stage network of decision units is considered. However, in this article, an attempt has been made to consider a network that includes all the involved sections, either series or in parallel, considering undesirable criteria such as percentage of waste per ton, the amount of mud produced per ton of stone, percentage of fines imposed on unlicensed workers, average machine repair time and so on.

In the studied articles, which use the principal component analysis approach to reduce the dimensions of the data, the public network has not been used as developed in this article. In this article, four sections including purchasing, production, support and sales are considered and in addition, in each section, all existing sub-networks affecting the efficiency of industrial units in the stone industry are considered. In addition, due to the fact that the stone industry in Iran, despite the richness of its mines has received less attention and researches have not talked about the efficiency of these industries, in this study we try to find a way to calculate the efficiency of stone industries so that the managers of this industry can identify their strengths and weaknesses.

3-Modeling and research problem

One of the most important approaches used in evaluating supply chain performance is the Network Data Envelopment Analysis model, which considers all the sectors involved in the efficiency and effectiveness of industrial units and based on the sub-criteria, calculates the efficiency of the network related to that industrial unit. In this research, this method has been used to calculate the efficiency of stone industrial units.

Since the number of criteria considered is large and their direct use in the DEA network method does not provide an acceptable result for the performance of the units, the principal component analysis method has been used to reduce the data size, in order to maintain a high percentage of data coverage variance, the volume of data used in the model can be significantly reduced.

Although the data that enters DEA cover a large variance of the data, not all of them, but however by reducing the dimensionality of the data, Principal Component Analysis speed up the computation. Furthermore, since the correlation between the data is lost by using principal component analysis, it can be expected that the accuracy of the model will increase.

After specifying the data obtained using the CCR approach of network data envelopment analysis, the efficiency of each subnet is calculated and finally the efficiency of the entire supply chain network for the factories is obtained. The proposed structure of the problem is as shown in figure 1.

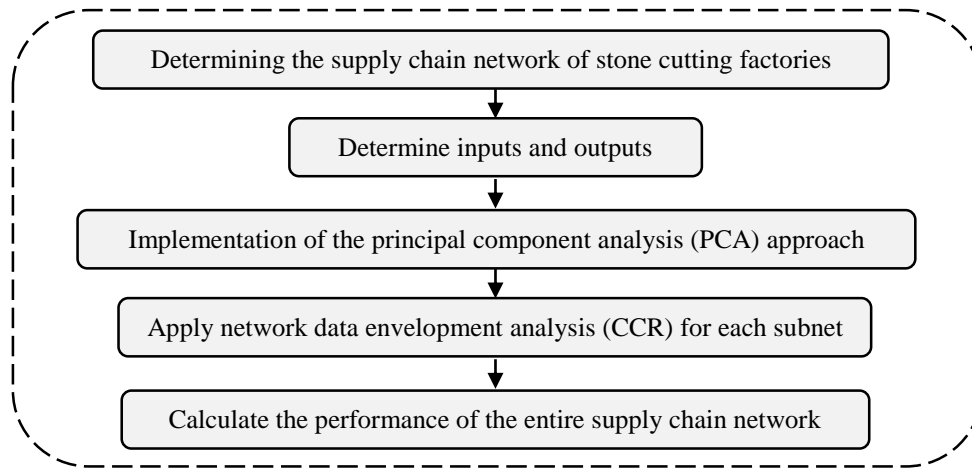


Fig 1. Proposed structure for calculating the efficiency of stone cutting factories

3-1-Using the Principal Component Analysis

In the data envelopment analysis method and in order to obtain the efficiency of each decision unit, the number of DMUs considered must be at least three times the number of input and output criteria. Since the number of criteria considered for evaluating this supply chain network is very large and requires examining the conditions of a large number of quarrying plants, it was tried to reduce the data dimensions by using the principal component analysis approach. In addition to maintaining a high percentage of data variance, the proposed model can be implemented and analyzed with a smaller number of decision units. Considering a problem with n variables, the i -th principal component is:

$$PC_i = a_{i1}x_1 + a_{i2}x_2 + \dots + a_{ij}x_j \quad (1)$$

If:

$i = 1, \dots, n$ (represents the i -th principal component)

$j = 1, \dots, n$ (represents the j -th principal variable)

a_{ij} = (the j -th component of the i -th vector of linear conversion factor, or eigenvector derived from the correlation matrix)

x_j = (the j -th principal variable.)

3-2-DEA and Network DEA

Data Envelopment Analysis is a mathematical programming model for evaluating the performance of decision-making units with multiple inputs and multiple outputs. This method uses mathematical programming techniques to evaluate the performance of a set of decision units. The first DEA model proposed by Charnes et al.(1978) is shown as statements (2) to (5). If there are n decision units in the problem and each of these units (DMU_j , $j = 1, 2, \dots, n$) uses m inputs (x_{ij} , $i = 1, \dots, m$) with weights v_i and s outputs (y_{rj} , $r = 1, \dots, s$) with weights u_r . The relative efficiency of the unit (DMU_o) is as follows. Where x_{io} and y_{ro} are the i -th input and r -th output of this DMU, respectively.

$$Max \sum_{r=1}^s u_r y_{ro} \quad (2)$$

$$\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, j = 1, \dots, n \quad (3)$$

$$\sum_{i=1}^m v_i x_{io} = 1 \quad (4)$$

$$u_r > 0, v_i > 0, r = 1, \dots, s, i = 1, \dots, m \quad (5)$$

In this paper, the supply chain network related to a number of stone factories was identified by studying their structure and considering the criteria involved in each of the subnets, all the effective factors in supply chain evaluation were determined. These criteria have been selected based on the SCOR model and include sustainability criteria and have been extracted using the judgment of industry experts and the use of stone strategic document (*Strategic Document for the Iranian Stone Industry, 2012*).

As mentioned, in this research, the network DEA approach has been used to evaluate the factories. The network intended for these factories is shown in figure 2. The first sub-network is related to the purchase of raw materials such as stone cutting and consumables such as resin, the second sub-network is related to the production process, the third sub-network is the support department including human resources management, accounting, finance, management and research & development, and finally the fourth sub-network is related to sales and delivery to customers.

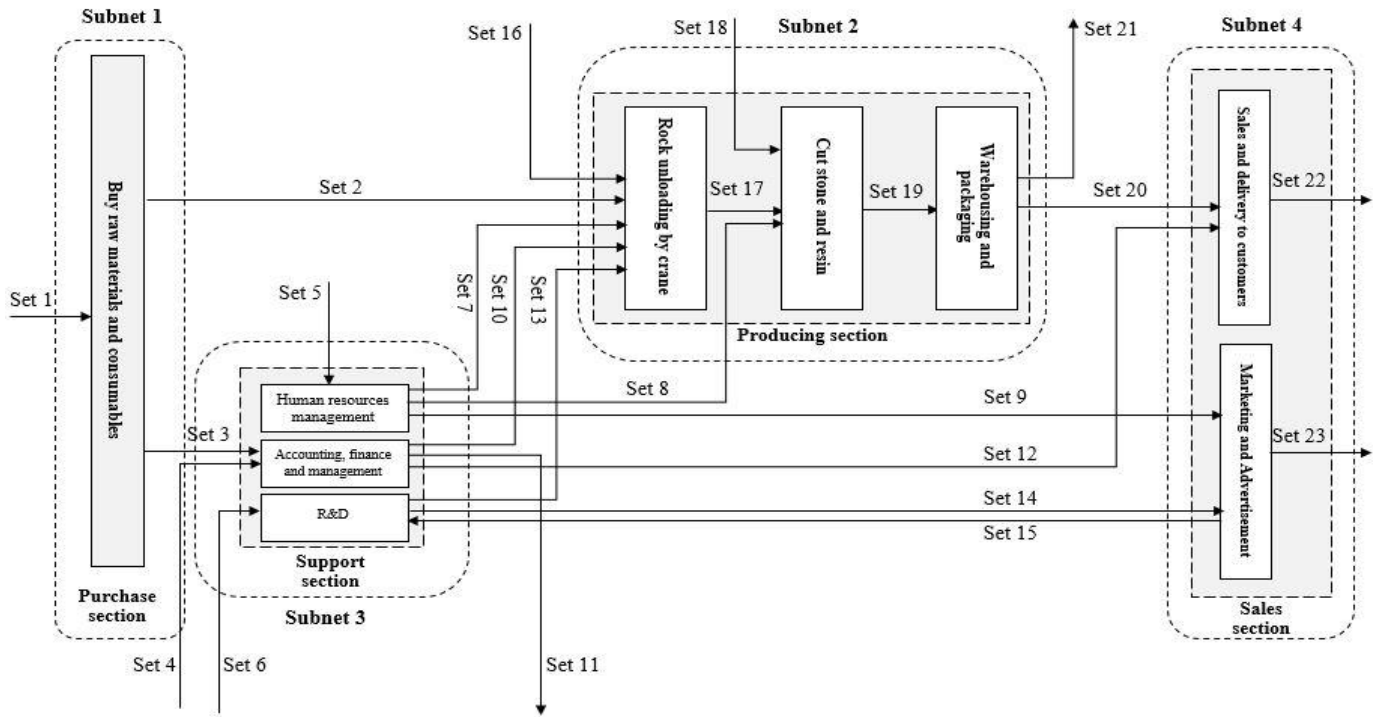


Fig 2. Supply chain network structure for stone factories

After determining the supply chain network of stone industrial units and communications between each subnet, the criteria involved in evaluating each subnet, which is the input and output of each section, were categorized according to table 1.

By specifying the data related to the input and output of the sub-networks in the main network of the supply chain, the CCR approach of data envelopment analysis is used and the efficiency of quarrying factories in all 4 sub-networks is obtained. The parameters used in these models are shown in table 2.

Table 1. Criteria involved in the supply chain network of stone factories

Sets	Criteria	Sets	Criteria		
Set 1	S1-1	Access level to stone mines	Set 9	S9-1	Introduction to the science of stone product marketing
	S1-2	Average access time to consumables including resin	Set 10	S10-1	Salary costs per person per hour
	S1-3	Time required to replace machinery and equipment		S10-2	Level of need for monitoring on the production line
	S1-4	The time required to adapt to the new mine in an emergency	Set 11	S11-1	The amount of liquidity as the ability of the unit to face the existing risks
	S1-5	Average distance between stone mines and industrial units	Set 12	S12-1	The share of the cost price of stone in the office sector
Set 2	S2-1	Percentage of waste per ton	Set 13	S13-1	Speed of applying new techniques in stone production and cutting
	S2-2	The amount of mud produced per ton of stone	Set 14	S14-1	Ability to meet the diverse needs of customers
Set 3	S3-1	Share of cost price of stone in raw materials and consumables		S14-2	The share of cost price of stone in marketing
	S3-2	Cost of raw materials and consumables such as resin and morsel	Set 15	S15-1	Get feedback from customers
	S3-3	Shipping cost per unit of rock cut per kilometer	S15-2	The level of knowledge of the target markets	
	S3-4	Considering the financial stability of mines related to the stone industrial unit	Set 16	S16-1	The cost of maintenance and repairs in each period
Set 4	S4-1	Percentage of receipts of checks and accounts received		S16-2	The amount of electricity consumed in each period in normal time
Set 5	S5-1	Number of non-productive employees	Set 17	S17-1	Precautionary storage of stone and consumables such as resin and morsels
	S5-2	Number of employees in the production sector	Set 18	S18-1	Average device repair time
	S5-3	Number of hours of formal training		S18-2	The rate of use of equipment after the end of their life
	S5-4	Number of hours of informal training while doing practical work		S18-3	The amount of water consumed in each period for each device
	S5-5	The amount of training of stone industry managers	Set 19	S19-1	Speed of cutting stone blocks and reaching the final product
	S5-6	Level of staff training in dealing with dangerous events		S19-2	The difference between the nominal capacity and the actual production
Set 6	S6-1	Factory land area	Set 20	S20-1	Percentage of closures of stone industrial units in each period
	S6-2	Familiarity with world trade rules for stone exports		S20-2	The cost of transporting each meter of stone per kilometer from the factory the customer
Set 7	S7-1	Number of shifts and working hours of stone industrial unit	S20-3	Ratio of goods exported abroad in the stone factory	
	S7-2	Ratio of indigenous workers to non-indigenous workers	S20-4	The share of the cost price of stone in the production sector	
	S7-3	Proportion of non-native workers without a license	Set 21	S21-1	The amount of stone stored at the end of each period
	S7-4	Percentage of fines imposed on unlicensed workers		S22-1	Time required to meet customer needs
	S7-5	Provide a safe environment for employees	Set 22	S22-2	The amount of direct sales of stone to consumers
S7-6	The level of workers' contact with environmental factors and pollution	S22-3		Intermediate sales of stone to brokers	
Set 8	S8-1	Level of expertise of human resources based on degree	Set 23	S23-1	Percentage change in sales due to marketing and advertising policies
	S8-2	Level of experience and work experience of human resources			

Table 2. Variables and parameters used in network data envelopment analysis model

x_{ij}	i -th input of the purchasing section for unit j	S_i	Weight of criterion x_{ij}	$I \in \{1, 2, \dots, i\}$
$y_{L_1j}^1$	L_1 -th output of the purchasing section for unit j	$v_{L_1}^1$	Weight of criterion $y_{L_1j}^1$	$L_1 \in \{1, 2, \dots, l_1\}$
$y_{L_2j}^2$	L_2 -th input of the human resources management section for unit j	$v_{L_2}^2$	Weight of criterion $y_{L_2j}^2$	$L_2 \in \{1, 2, \dots, l_2\}$
$y_{L_3j}^3$	L_3 -th input of accounting, finance and management section for unit j	$v_{L_3}^3$	Weight of criterion $y_{L_3j}^3$	$L_3 \in \{1, 2, \dots, l_3\}$
$y_{L_4j}^4$	L_4 -th input of the research and development section for unit j	$v_{L_4}^4$	Weight of criterion $y_{L_4j}^4$	$L_4 \in \{1, 2, \dots, l_4\}$
$Z_{f_1j}^1$	F_1 -th output of the human resources management section for unit j	$w_{f_1}^1$	Weight of criterion $Z_{f_1j}^1$	$F_1 \in \{1, 2, \dots, f_1\}$
$Z_{f_2j}^2$	F_2 -th output of accounting, finance and management section for unit j	$w_{f_2}^2$	Weight of criterion $Z_{f_2j}^2$	$F_2 \in \{1, 2, \dots, f_2\}$
$Z_{f_3j}^3$	F_3 -th output of the R&D section for unit j	$w_{f_3}^3$	Weight of criterion $Z_{f_3j}^3$	$F_3 \in \{1, 2, \dots, f_3\}$
$Z_{u_1j}^4$	U_1 -th input of the rock discharge section for unit j	$w_{u_1}^4$	Weight of criterion $Z_{u_1j}^4$	$U_1 \in \{1, 2, \dots, u_1\}$
$Z_{u_2j}^5$	U_2 -th output of the rock discharge section and the input of the rock cutting section for unit j	$w_{u_2}^5$	Weight of criterion $Z_{u_2j}^5$	$U_2 \in \{1, 2, \dots, u_2\}$
$Z_{u_3j}^6$	U_3 -th input of the rock cutting section for unit j	$w_{u_3}^6$	Weight of criterion $Z_{u_3j}^6$	$U_3 \in \{1, 2, \dots, u_3\}$
$Z_{u_4j}^7$	U_4 -th output of the rock cutting section and the input of the warehousing and packaging section for unit j	$w_{u_4}^7$	Weight of criterion $Z_{u_4j}^7$	$U_4 \in \{1, 2, \dots, u_4\}$
$T_{u_5j}^1$	U_5 -th output of the warehousing and packaging section for unit j	$H_{u_5}^1$	Weight of criterion $T_{u_5j}^1$	$U_5 \in \{1, 2, \dots, u_5\}$
$T_{R_2j}^2$	R_2 -th input of the delivery to customers section for unit j	$H_{R_2}^2$	Weight of criterion $T_{R_2j}^2$	$R_2 \in \{1, 2, \dots, r_2\}$
$T_{R_3j}^3$	R_3 -th input of the marketing and advertisement section for unit j	$H_{R_3}^3$	Weight of criterion $T_{R_3j}^3$	$R_3 \in \{1, 2, \dots, r_3\}$
$P_{S_1j}^1$	S_1 -th output of the delivery to customers section for unit j	$E_{S_1}^1$	Weight of criterion $P_{S_1j}^1$	$S_1 \in \{1, 2, \dots, s_1\}$
$P_{S_2j}^2$	S_2 -th output of the marketing and advertisement section for unit j	$E_{S_2}^2$	Weight of criterion $P_{S_2j}^2$	$S_2 \in \{1, 2, \dots, s_2\}$

The data envelopment analysis model for the purchasing of raw materials and consumables, considering the CCR model, is in the form of statements (6) to (9).

$$\text{Max} \sum_{L_1} v_{L_1}^1 y_{L_1k}^1 \quad (6)$$

$$\sum_I s_I x_{Ik} = \nu \quad (7)$$

$$\sum_{L_1} v_{L_1}^1 y_{L_1j}^1 - \sum_I s_I x_{Ij} \leq 0 \quad \forall j \in \{1, 2, \dots, N\} \quad (8)$$

$$v_{L_1}^1, s_I \geq \varepsilon \quad \forall I \in \{1, 2, \dots, i\}, \quad \forall L_1 \in \{1, 2, \dots, l_1\} \quad (9)$$

The data envelopment analysis model for the support department, considering the CCR model, is shown as statements (10) to (16).

$$\text{Max} \sum_{F_1} z_{F_1k}^1 w_{F_1}^1 + \sum_{F_2} z_{F_2k}^2 w_{F_2}^2 + \sum_{F_3} z_{F_3k}^3 w_{F_3}^3 \quad (10)$$

$$\sum_{L_2} y_{L_2k}^2 v_{L_2}^2 + \sum_{L_3} y_{L_3k}^3 v_{L_3}^3 + \sum_{L_4} y_{L_4k}^4 v_{L_4}^4 = \nu \quad (11)$$

$$\sum_{F_1} z_{F_1j}^1 w_{F_1}^1 + \sum_{F_2} z_{F_2j}^2 w_{F_2}^2 + \sum_{F_3} z_{F_3j}^3 w_{F_3}^3 - \left[\sum_{L_2} y_{L_2j}^2 v_{L_2}^2 + \sum_{L_3} y_{L_3j}^3 v_{L_3}^3 + \sum_{L_4} y_{L_4j}^4 v_{L_4}^4 \right] \leq 0 \quad \forall j \in \{1, 2, \dots, N\} \quad (12)$$

$$\sum_{F_1} z_{F_1j}^1 w_{F_1}^1 - \sum_{L_2} y_{L_2j}^2 v_{L_2}^2 \leq 0 \quad \forall j \in \{1, 2, \dots, N\} \quad (13)$$

$$\sum_{F_2} z_{F_2j}^2 w_{F_2}^2 - \sum_{L_3} y_{L_3j}^3 v_{L_3}^3 \leq 0 \quad \forall j \in \{1,2, \dots, N\} \quad (14)$$

$$\sum_{F_3} z_{F_3j}^3 w_{F_3}^3 - \sum_{L_4} y_{L_4j}^4 v_{L_4}^4 \leq 0 \quad \forall j \in \{1,2, \dots, N\} \quad (15)$$

$$v_{L_2}^2, v_{L_3}^3, v_{L_4}^4, w_{F_1}^1, w_{F_2}^2, w_{F_3}^3 \geq \varepsilon \quad \forall L_2 \in \{1,2, \dots, l_2\}, \quad (16)$$

$$\forall L_3 \in \{1,2, \dots, l_3\}, \forall L_4 \in \{1,2, \dots, l_4\}, \forall F_1 \in \{1,2, \dots, f_1\},$$

The data envelopment analysis model for the production sector, considering the CCR model, is shown as statements (17) to (23).

$$Max \sum_{U_5} T_{U_5k}^1 H_{U_5}^1 \quad (17)$$

$$\sum_{U_1} z_{U_1k}^4 w_{U_1}^4 + \sum_{U_3} z_{U_3k}^6 w_{U_3}^6 = 1 \quad (18)$$

$$\sum_{U_5} T_{U_5j}^1 H_{U_5}^1 - \left[\sum_{U_1} z_{U_1j}^4 w_{U_1}^4 + \sum_{U_3} z_{U_3j}^6 w_{U_3}^6 \right] \leq 0 \quad \forall j \in \{1,2, \dots, N\} \quad (19)$$

$$\sum_{U_2} z_{U_2j}^5 w_{U_2}^5 - \sum_{U_1} z_{U_1j}^4 w_{U_1}^4 \leq 0 \quad \forall j \in \{1,2, \dots, N\} \quad (20)$$

$$\sum_{U_4} z_{U_4j}^7 w_{U_4}^7 - \left[\sum_{U_2} z_{U_2j}^5 w_{U_2}^5 + \sum_{U_3} z_{U_3j}^6 w_{U_3}^6 \right] \leq 0 \quad \forall j \in \{1,2, \dots, N\} \quad (21)$$

$$\sum_{U_5} T_{U_5j}^1 H_{U_5}^1 - \sum_{U_4} z_{U_4j}^7 w_{U_4}^7 \leq 0 \quad \forall j \in \{1,2, \dots, N\} \quad (22)$$

$$w_{U_1}^4, w_{U_2}^5, w_{U_3}^6, w_{U_4}^7, H_{U_5}^1 \geq \varepsilon \quad \forall U_1 \in \{1,2, \dots, u_1\}, \quad (23)$$

$$\forall U_2 \in \{1,2, \dots, u_2\}, \forall U_3 \in \{1,2, \dots, u_3\}, \forall U_4 \in \{1,2, \dots, u_4\}, \forall U_5 \in \{1,2, \dots, u_5\}$$

The data envelopment analysis model for the sales department, considering the CCR model, is shown as statements (24) to (29).

$$Max \sum_{S_1} P_{S_1k}^1 E_{S_1}^1 + \sum_{S_2} P_{S_2k}^2 E_{S_2}^2 \quad (24)$$

$$\sum_{R_2} T_{R_2k}^2 H_{R_2}^2 + \sum_{R_3} T_{R_3k}^3 H_{R_3}^3 = 1 \quad (25)$$

$$\sum_{S_1} P_{S_1j}^1 E_{S_1}^1 + \sum_{S_2} P_{S_2j}^2 E_{S_2}^2 - \left[\sum_{R_2} T_{R_2j}^2 H_{R_2}^2 + \sum_{R_3} T_{R_3j}^3 H_{R_3}^3 \right] \leq 0 \quad \forall j \in \{1,2, \dots, N\} \quad (26)$$

$$\sum_{S_1} P_{S_1j}^1 E_{S_1}^1 - \sum_{R_2} T_{R_2j}^2 H_{R_2}^2 \leq 0 \quad \forall j \in \{1,2, \dots, N\} \quad (27)$$

$$\sum_{S_2} P_{S_2j}^2 E_{S_2}^2 - \sum_{R_3} T_{R_3j}^3 H_{R_3}^3 \leq 0 \quad \forall j \in \{1,2, \dots, N\} \quad (28)$$

$$H_{R_2}^2, H_{R_3}^3, E_{S_1}^1, E_{S_2}^2 \geq \varepsilon \quad \forall R_2 \in \{1,2, \dots, r_2\}, \forall R_3 \in \{1,2, \dots, r_3\}, \forall S_1 \in \{1,2, \dots, s_1\}, \forall S_2 \in \{1,2, \dots, s_2\} \quad (29)$$

4-Model implementation

To conduct this research in the cities of Tehran (Shams Abad Industrial Town), Isfahan (Mahmoud Abad Industrial Town) and Qom (Mahmudabad Industrial Towns and Sang Omid Town), 26 stone cutting factories that have been placed at the same level based on stone quality and production quantity with respect to industrial unit capacity, were considered and related data were collected. The supply chain network of each factory was determined according to the administrative, support, production and sales processes of the factory, and the input and output criteria of each stage were collected using the stone strategic document and the industry experts' opinions. The principal component analysis approach was implemented in the

inputs and outputs of each section in MATLAB software. In this approach, the main components that cover a high percentage of the total variance of the data are selected in each section. The amount of variance covered by these principal components is shown in Table 3. According to table 1 on the inputs and outputs of the network data envelopment analysis subnet networks, the data of 26 factories were collected and presented in table 4.

Table 3. The amount of variance covered by the selected principal components

Ad output	Ad input	Delivery to customer output	Delivery to customer input	Warehouse output	Cutting output	Cutting input	Discharge output	Discharge input	R&D output	R&D input	Financial management output	Financial management input	Human resource output	Human resource input	Purchase output	Purchase input	
3	3	3	5	5	2	5	1	12	3	4	4	5	9	6	6	5	Number of criteria
2	2	2	1	2	1	1	1	2	1	1	2	1	2	2	1	2	Number of principal components
0.837137	0.933966	0.923713	0.840984	0.787809	0.716383	0.758984	1.000000	0.776797	0.679688	0.632955	0.780974	0.819278	0.679027	0.883025	0.808358	0.758368	explained variance

Table 4. Data related to stone industrial units for use in the model

DMU26	DMU25	DMU24	DMU23	DMU22	DMU21	DMU20	DMU19	DMU18	DMU17	DMU16	DMU15	DMU14	DMU13	DMU12	DMU11	DMU10	DMU9	DMU8	DMU7	DMU6	DMU5	DMU4	DMU3	DMU2	DMU1	Factors
0.4	1.5	1	1	0.3	2	2	1	1	0.5	0.6	0.5	1	0.4	3	2	0.7	1	0.5	0.5	1	1	1.5	1	1	2	S1-1
1	1	2	1	1	3	2	1	2	1	1	1	1.5	1	2	1	4	2	1	0	3	2	0	1	1	1	S1-2
2	0.4	8	2	4	4	6	3	2	2	4	3	4	5	10	4	1.5	2	1	1	8	4	1	0.5	0.5	1	S1-3
1	0.5	2	2	1	2	2	3	1	1	2	1	0.5	0.2	0.5	0.1	1	1	0	5	0.5	6	0.8	0.5	2	0.5	S1-4
350	550	550	300	400	160	150	600	140	140	180	200	380	400	200	350	250	500	100	135	400	300	300	700	500	200	S1-5
30	40	35	25	30	50	50	35	40	40	35	30	30	30	35	30	25	30	25	28	30	35	35	50	50	30	S2-1
50	180	50	150	60	35	40	50	40	50	60	60	70	50	150	120	200	80	130	50	100	130	100	500	250	150	S2-2
70	75	80	90	75	60	60	70	50	55	60	70	70	70	60	75	70	85	70	80	70	75	70	80	80	90	S3-1
800	700	800	1200	800	200	300	500	200	200	300	700	480	700	500	300	480	500	950	300	250	450	750	800	950	900	S3-2
400	550	600	350	400	300	250	600	250	220	350	350	350	300	300	200	220	340	225	400	350	250	300	500	750	500	S3-3
830	900	4700	20000	9000	890	4500	8000	1400	950	3400	4500	2000	4000	500	1600	850	1300	900	1300	7000	3500	1500	500	20000	15000	S3-6
70	80	70	80	85	75	70	80	70	75	85	90	85	80	70	60	70	70	50	75	70	100	80	90	90	80	S4-1
3	6	2	4	3	2	1	2	1	1	1	2	2	4	2	4	4	4	2	1	2	4	8	5	15	4	S5-1
9	15	8	20	8	3	4	5	2	4	3	7	10	15	8	10	13	10	15	11	10	20	15	13	40	19	S5-2
6	0	1	4	2	0	0	0	0	0	0	2	0	2	0	5	0	0	1	0	0	7	5	0	10	0	S5-3
3	0	7	7	5	2	1.5	1	3	2	1	4	5	10	5	8	15	10	10	5	10	10	5	0	5	2	S5-4
2	3	2	3	2	3	3	3	3	1	2	3	3	1	2	2	2	3	2	3	1	2	2	2	1	2	S5-5
4	4	4	3	2	2	3	3	5	5	4	3	2	1	4	2	3	2	4	3	3	5	4	4	3	4	S5-6
5	5	10	11	6	4.2	3.9	4	3.7	3.5	3.8	4	4.5	7	6	3.8	5	6	6	6.5	11	6.5	7	5	8	5	S6-1
2	2	3	4	3	1	1	2	2	2	2	3	1	2	1	1	1	2	1	3	1	5	3	1	1	3	S6-2
12	12	12	20	10	8	8	12	8	12	10	12	10	8	8	8	8	10	12	10	10	12	8	11	12	10	S7-1
70	30	100	50	90	85	90	100	100	100	100	90	80	50	60	65	40	50	15	50	20	90	100	20	65	95	S7-2
2	10	3	2	0	2	2	1	1	2	2	0	5	10	8	5	0	19	8	3	8	0	0	13	35	0	S7-3
0	0	5	0	0	4	0	0	6	0	0	5	5	5	0	0	0	0	6	0	5	0	0	0	0	0	S7-4
4	3	4	4	4	3	3	4	3	3	3	4	3	4	4	3	3	2	4	3	3	4	4	3	4	3	S7-5
2	3	3	3	3	3	3	3	3	3	3	3	3	3	3	4	4	3	4	4	3	4	3	4	2	3	S7-6
2	2	2	3	2	2	2	2	2	2	2	2	1	3	2	3	3	2	1	3	3	3	2	1	2	3	S8-1
12	10	15	11	10	4	4	7	2	3	6	15	8	15	10	6	5	7	8	10	15	15	2	10	5	8	S8-2
3	1	3	3	3	1	1	2	1	1	1	3	2	4	3	3	4	1	1	3	3	5	3	1	1	3	S9-1
13000	8000	14000	17000	15000	12000	12000	14000	10000	11000	10000	15000	10000	12000	9000	7000	9500	8000	5000	10000	6000	7500	7500	8000	10000	9000	S10-1
100	100	85	90	100	70	60	80	60	40	50	100	80	100	80	100	80	100	100	100	80	50	70	100	100	90	S10-2
100	500	70	200	100	0	150	50	0	0	200	100	200	500	0	400	300	200	700	200	300	200	500	500	500	0	S11-1
10	10	5	2.5	10	10	10	15	5	10	10	15	10	5	10	5	10	5	10	7	10	10	10	10	10	2.5	S12-1
2	1	2	4	3	2	2	2	2	2	1	1	2	4	2	3	3	1	5	1	1	3	3	1	3	4	S13-1
12	20	7	14	10	15	12	11	9	11	10	14	10	4	7	3	7	14	60	90	90	30	15	30	12	15	S14-1
10	5	0	2.5	5	0	0	0	0	0	10	5	10	5	10	5	0	0	0	3	0	5	5	5	0	2.5	S14-2
3	2	3	4	3	2	3	3	1	4	4	5	2	3	4	4	3	2	4	3	3	4	4	4	3	5	S15-1
3	2	3	3	2	2	1	1	1	1	1	3	3	4	2	2	4	4	4	3	3	4	3	1	1	3	S15-2
4.5	6	4	11	7	2	2.5	4.5	2.5	2.5	3.5	5.5	4	6	7	11	20	5.5	12	8	5	7.5	4	3.5	6	4	S16-1
3.8	15	3	10	3.5	1.4	1.7	2.5	1.5	1.9	2	3.2	8	7.8	8.7	9	3	4.5	5.5	8	3.5	6	12	15	25	10	S16-2
2	0	2	1.25	1.5	1.5	2	2	2.5	1.5	1.75	1.5	2	2	0	3	2	0	1.5	2	1.5	2	2	0	3	2	S17-1
24	48	48	48	24	48	48	36	48	36	24	24	48	12	24	48	12	12	48	24	48	0	24	2	48	2	S18-1
1	3	2	1	2	3	3	2	2	2	1	2	1	1	4	2	1	1	1	2	2	3	1	2	2	1	S18-2
4	2.8	3.8	4	4	6	3.5	4.8	5.3	5	4	3	3	2.9	4.5	4	3	0.5	3.5	3	3	8	3	2.8	4	3	S18-3
12.5	22	14	35	15	8	9	11	10	12	12	11	20	37	12.5	20	15	15	17	10	12	20	22	25	22	40	S19-1
2.5	4	2	2	3	3.5	2	2.5	3	2.5	3	2	2	1	3	4	2.4	4.5	3	0.5	0	3	0	4	0	5	S19-2
0	5	3	0	4	0	2	1	0	7	2	0	0	0	5	3	0	4	0	2	1	0	7	2	0	0	S20-1
30	22	18	28	30	30	25	10	15	30	55	35	25	30	22	18	28	30	30	25	10	15	30	55	35	25	S20-2
0	0	0	0	0	0	0	0	90	0	0	30	15	0	0	0	0	0	0	0	0	90	0	0	30	15	S20-3
20	20	15	20	10	20	10	20	10	15	5	10	5	20	20	15	20	10	20	10	20	10	15	5	10	5	S20-4
3500	2000	3000	4000	2500	500	500	1500	1000	200	500	2000	2000	2000	1500	2000	1000	1000	1300	3000	3000	1500	2000	1800	2000	2000	S21-1
2	2	3	1	2	5	4	2	2	3	2	2	4	5	7	3	1	1	3	3	15	45	5	2	1	1	S22-1
1.5	2.5	2	5	1.5	0.2	0.6	0.8	0.5	0.4	2	1.2	3	5	1.5	2.8	3.24	4.5	1.8	2.1	3.2	5	4	2	1.6	2	S22-2
1.5	5.5	1.5	2	2	1.1	0.9	1.2	1.5	1.6	0.8	1.5	1.8	4	1.5	2	0.36	0	6.4	0.9	0.8	1	1.2	5.5	6.4	8	S22-3
30	40	70	60	40	50	35	20	50	40	50	75	30	90	50	40	45	80	20	30	60	30	40	40	50	70	S23-1

The values of the selected principal components that are the final inputs and outputs in each data coverage analysis subnet are also shown in table 5.

Table 5. Principal component values in each part of the supply chain network

DMUS	O_i		M_{O_h, I_i}		I_h	M_{O_g, I_h}		I_g	O_f		I_f		O_e		I_e	O_d		I_d	O_c		I_c	O_b		I_b		O_a			I_a																																																																																																																																																																																																																																																																																																										
	1	2	1	2	1	1	2	1	2	1	2	1	2	1	1	2	1	1	2	1	2	1	1	2	1	2	1	2	3																																																																																																																																																																																																																																																																																																										
DMU1	-0.85902	0.41907	-0.70544	2.2593	5.11516	-1.44491	-0.89987	-1.77784	0.52358	-1.75419	-0.1712	1.14481	-1.85214	2.38075	3.00483	-0.5264	-3.88071	6.91684	0.83957	0.08898	0.91573	-1.78271	0.8985	-0.79624	-2.50504	1.03713	2.41305	-1.59547	2.41305	-0.54597	2.52451	2.04992	-0.51253	-1.98089	0.10602	-1.53974	-0.46237	1.25208	0.20691	3.75277	-1.71428	6.45263	-6.71991	-6.22376	-1.53621	0.81733	1.32026	3.29914	2.72189	-1.49906	-6.62746	0.3654	1.3654	-1.6346	0.3654	-0.1346	0.3654	-1.6346	0.3654	2.09067	5.19999	-0.06171	-0.06171	7.55146	-2.10383	0.58487	-1.46517	-0.83264	0.94583	2.08343	0.06493	0.81004	2.3318	0.07327	1.32672	0.05659	-0.19178	0.20906	2.40226	-0.13377	-1.04794	-0.55355	-1.27386	1.32812	-1.57437	-2.24135	3.45699	0.12407	-0.80075	0.16675	4.47781	0.64957	-1.63927	1.13979	-0.13698	1.50939	6.14571	6.14571	6.76082	3.03777	-1.4444	-1.84927	-1.30624	0.10869	-1.11641	0.68851	3.22271	-1.26297	-1.74572	-1.5905	4.71765	1.79657	4.24345	3.26524	2.87073	-2.0009	-6.78429	-2.06152	0.25072	-0.31417	-0.00847	1.4865	3.54669	0.9614	-0.41389	-0.06717	1.71157	0.55308	0.37919	-0.80318	2.76073	-5.66948	-4.12	-3.18249	1.28624	1.9866	1.71467	1.39379	-4.08155	2.47818	2.00947	0.36632	0.78735	-1.39868	-11.6567	-6.64057	-11.0848	-0.72441	-7.13102	3.47965	-6.30375	3.44897	-6.09462	3.44091	-1.30189	3.97316	3.365	-0.81592	-3.15078	1.4262	1.66506	1.34424	-4.73708	-1.80908	-4.22936	-3.1116	-3.04384	1.92171	6.75714	1.85934	0.37723	-1.4802	1.31436	-1.47023	-1.63504	-4.82558	-0.94997	-0.3889	-0.48862	0.6049	2.19881	0.11521	-1.48243	5.38637	-1.72628	-2.59602	-3.06545	-2.56094	-3.18542	-3.18542	1.55049	-4.34897	2.41693	-1.66338	1.53373	-1.20831	3.48391	3.24877	0.9031	2.46531	2.51756	1.14078	3.03448	1.19029	5.57611	3.71145	0.30062	5.26005	-0.46007	1.73828	-111.139	-162.29	27.9827	24.3436	13.8839	-11.7637	38.0792	26.5004	32.4894	40.3645	39.2396	42.971	2.50087	2.13983	-2.30007	-4.86041	1.32743	3.57662	-2.26104	-1.55321	-4.71281	-2.94359	-3.32683	-1.0303	-0.50188	0.90854	-1.49044	-3.10982	2.30606	-7.55126	5.79477	7.63841	1.54467	1.82228	-1.51812	-3.07525	-2.99693	0.77615	6.51672	1.68478	11.4662	3.4658	4.57506	0.09923	-4.65053	-2.1778	-4.08245	-3.53667	-6.53577	0.10768	-1.52814	-2.53736	2.96814	-8.84865	4.17315	-3.21132	-6.29579	-2.39403	2.02889	-2.77026	-3.14271	-7.06214	-3.90649	1.63337	-4.31402	11.1236	16.2048	-2.76199	-2.45559	-1.51857	1.21219	-3.81844	-2.48081	-3.2192	-3.96816	-3.82715	-4.25005	-0.2829	-1.36132	0.90321	-0.03152	-0.73579	4.3605	-0.53355	2.96752	-1.97122	-0.03146	-0.74712	-1.22808	-1.07483	-0.82613	-1.03403	3.79334	3.79334	-0.11103	-1.38653	1.12863	-2.76847	-1.83314	1.76571	-0.70588	0.6275	-0.74905	1.15874	-2.09787	-2.8322	-2.87397	-2.35363	0.75065	4.91394	-2.48395	-2.20417	-1.17862	-1.28476	0.79741	6.9917	1.59382
Legend	I : Inputs			a : Purchase						d : Research and Development						g : Drain the stone																																																																																																																																																																																																																																																																																																																							
	O : Outputs			b : Human Resources						e : Delivery to customers						h : stone cutting																																																																																																																																																																																																																																																																																																																							
	M : Middle			c : Accounting and Finance						f : Advertise						i : Store																																																																																																																																																																																																																																																																																																																							

Table 5. Continued

DMUS	I_a	O_a	I_b	O_b	I_c	O_c	I_d	O_d	I_e	O_e	I_f	O_f	I_g	$M_{O_g I_h}$	I_h	$M_{O_h I_i}$	O_i		
																	1	2	
DMU14	0.75641	-0.30067	0.67645	-1.25929	-1.04654	-0.4384	2.95357	1.15023	0.58611	-1.28789	4.66572	-2.84748	-1.34657					-0.75684	-1.58244
DMU15	0.82447	-1.05414	-1.47186	-1.66983	-1.70624	2.27276	-1.74489	-1.74706	0.91649	-0.39808	2.21061	1.84891	0.35785					-0.24102	1.62285
DMU16	0.75641	-0.30067	0.67645	-1.25929	-1.04654	-0.4384	2.95357	1.15023	0.58611	-1.28789	4.66572	-2.84748	-1.34657					-0.75684	-1.58244
DMU17	0.82447	-1.05414	-1.47186	-1.66983	-1.70624	2.27276	-1.74489	-1.74706	0.91649	-0.39808	2.21061	1.84891	0.35785					-0.24102	1.62285
DMU18	0.75641	-0.30067	0.67645	-1.25929	-1.04654	-0.4384	2.95357	1.15023	0.58611	-1.28789	4.66572	-2.84748	-1.34657					-0.75684	-1.58244
DMU19	0.82447	-1.05414	-1.47186	-1.66983	-1.70624	2.27276	-1.74489	-1.74706	0.91649	-0.39808	2.21061	1.84891	0.35785					-0.24102	1.62285
DMU20	0.75641	-0.30067	0.67645	-1.25929	-1.04654	-0.4384	2.95357	1.15023	0.58611	-1.28789	4.66572	-2.84748	-1.34657					-0.75684	-1.58244
DMU21	0.82447	-1.05414	-1.47186	-1.66983	-1.70624	2.27276	-1.74489	-1.74706	0.91649	-0.39808	2.21061	1.84891	0.35785					-0.24102	1.62285
DMU22	0.75641	-0.30067	0.67645	-1.25929	-1.04654	-0.4384	2.95357	1.15023	0.58611	-1.28789	4.66572	-2.84748	-1.34657					-0.75684	-1.58244
DMU23	0.82447	-1.05414	-1.47186	-1.66983	-1.70624	2.27276	-1.74489	-1.74706	0.91649	-0.39808	2.21061	1.84891	0.35785					-0.24102	1.62285
DMU24	0.75641	-0.30067	0.67645	-1.25929	-1.04654	-0.4384	2.95357	1.15023	0.58611	-1.28789	4.66572	-2.84748	-1.34657					-0.75684	-1.58244
DMU25	0.82447	-1.05414	-1.47186	-1.66983	-1.70624	2.27276	-1.74489	-1.74706	0.91649	-0.39808	2.21061	1.84891	0.35785					-0.24102	1.62285
DMU26	0.75641	-0.30067	0.67645	-1.25929	-1.04654	-0.4384	2.95357	1.15023	0.58611	-1.28789	4.66572	-2.84748	-1.34657					-0.75684	-1.58244

Legend	I : Inputs	a : Purchase	d : Research and Development	g : Drain the stone
	O : Outputs	b : Human Resources	e : Delivery to customers	h : stone cutting
	M : Middle	c : Accounting and Finance	f : Advertise	i : Store

Because negative data cannot be used in the data envelopment analysis method, using equations 30 and 31, all the numbers related to the considered principal components will be converted to non-negative numbers.

In these equations, based on the values of PC_j as the principal components, the value of the parameter Q is calculated based on equation 31, and then the value of Q obtained is added to the value of the principal component of the sum and the parameter Z_j is obtained.

$$Z_j = PC_j + Q$$

$$Q = -\min\{PC_j\} + 1 \quad (31)$$

In addition to the above issues, because each principal component covers a part of the total data variance, the importance of each component will be different from the other component. In other words, the greater the variance that a component covers, the greater the value of that component. Accordingly, the values of input and output criteria based on this value will be entered in the data envelopment analysis method. This value is calculated based on the ratio of the variance covered by each component to the sum of the total variance covered by the selected components. First, by considering the variance of each component, the weight of each component is calculated as a criterion in data envelopment analysis. Second, the weight obtained for each criterion is multiplied by the column related to the non-negative numbers of that criterion and the weight criteria are considered as the final data to enter the data envelopment analysis approach. The final results will be shown in the table 6.

Table 6. Weighted and non-negative principal component values in each part of the supply chain network

DMUS	O_i		M_{O_h, I_i}		I_h	M_{O_g, I_h}		I_g	O_f		I_f		O_e		I_e	O_d		I_d	O_c		I_c	O_b		I_b			O_a			I_a									
	1	2	1	2	1	1	2	1	1	2	1	2	1	1	2	1	1	2	1	1	2	1	1	2	1	2	3	1	2	3									
DMU1	0.6355	1.0133	0.6809	1.5573	2.4016	0.4623	0.6234	0.3639	1.0442	0.3709	0.8388	1.2279	0.3419	5.4357	5.8753	3.3880	1.0253	8.6309	4.3501	3.8214	4.4038	2.5030	4.3916	3.1979	1.9943	4.4893	0.6355	1.0133	0.6809	1.5573	2.4016	0.4623	0.6234	0.3639	1.0442	0.3709	0.8388	1.2279	0.3419
DMU2	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899	0.6603	1.2663	1.8261	0.8832	0.5752	1.1826	0.2415	0.4674	1.3364	0.7396	1.0616	0.7292	1.1899	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899
DMU3	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899	0.6603	1.2663	1.8261	0.8832	0.5752	1.1826	0.2415	0.4674	1.3364	0.7396	1.0616	0.7292	1.1899	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899
DMU4	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899	0.6603	1.2663	1.8261	0.8832	0.5752	1.1826	0.2415	0.4674	1.3364	0.7396	1.0616	0.7292	1.1899	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899
DMU5	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899	0.6603	1.2663	1.8261	0.8832	0.5752	1.1826	0.2415	0.4674	1.3364	0.7396	1.0616	0.7292	1.1899	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899
DMU6	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899	0.6603	1.2663	1.8261	0.8832	0.5752	1.1826	0.2415	0.4674	1.3364	0.7396	1.0616	0.7292	1.1899	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899
DMU7	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899	0.6603	1.2663	1.8261	0.8832	0.5752	1.1826	0.2415	0.4674	1.3364	0.7396	1.0616	0.7292	1.1899	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899
DMU8	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899	0.6603	1.2663	1.8261	0.8832	0.5752	1.1826	0.2415	0.4674	1.3364	0.7396	1.0616	0.7292	1.1899	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899
DMU9	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899	0.6603	1.2663	1.8261	0.8832	0.5752	1.1826	0.2415	0.4674	1.3364	0.7396	1.0616	0.7292	1.1899	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899
DMU10	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899	0.6603	1.2663	1.8261	0.8832	0.5752	1.1826	0.2415	0.4674	1.3364	0.7396	1.0616	0.7292	1.1899	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899
DMU11	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899	0.6603	1.2663	1.8261	0.8832	0.5752	1.1826	0.2415	0.4674	1.3364	0.7396	1.0616	0.7292	1.1899	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899
DMU12	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899	0.6603	1.2663	1.8261	0.8832	0.5752	1.1826	0.2415	0.4674	1.3364	0.7396	1.0616	0.7292	1.1899	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899
DMU13	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899	0.6603	1.2663	1.8261	0.8832	0.5752	1.1826	0.2415	0.4674	1.3364	0.7396	1.0616	0.7292	1.1899	0.6085	0.5841	0.8880	2.7012	5.1329	0.8119	0.9753	1.5743	1.5123	2.7285	6.3465	3.1937	1.899

Legend
 I : Inputs
 O : Outputs
 M : Middle
 a : Purchase
 b : Human Resources
 c : Accounting and Finance
 d : Research and Development
 e : Delivery to customers
 f : Advertise
 g : Drain the stone
 h : stone cutting
 i : Store

The results of the implementation of four data envelopment analysis models in GAMS software in order to find each section performance are shown in table 7.

Table 7. Efficiency of each subnet for stone industrial units – proposed model (NDEA and PCA)

DMUs	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10	DMU11	DMU12	DMU13	DMU14	DMU15	DMU16	DMU17	DMU18	DMU19	DMU20	DMU21	DMU22	DMU23	DMU24	DMU25	DMU26
Purchase	1.000000	1.000000	0.176522	0.192135	0.278559	0.276936	0.318786	0.326759	0.097497	0.101197	0.114607	0.089381	0.24897	0.132339	0.351183	0.249824	0.071575	0.102865	0.258274	0.349109	0.080652	0.36902	1.000000	0.186719	0.199462	0.125023
Support	0.476755	1.000000	0.29827	0.460705	1.000000	1.000000	0.401688	0.642952	0.450823	1.000000	0.440955	0.427878	1.000000	0.576216	0.711361	0.455538	0.426595	0.54642	0.461843	0.397841	0.418554	0.604461	1.000000	1.000000	0.298088	0.423349
Producing	0.571542	0.181729	1.000000	0.581313	1.000000	0.383712	0.471282	0.332283	0.226737	1.000000	0.266789	0.314248	0.406789	0.385136	1.000000	0.688768	0.782405	1.000000	0.89003	0.889498	0.961581	0.927101	0.732959	0.947305	0.228893	0.677459
Sale	0.780803	0.513912	0.388397	0.863028	0.802007	0.258819	0.306507	0.551223	0.894393	1.000000	0.932001	1.000000	1.000000	0.977285	0.821856	0.864414	0.550895	0.593501	0.517827	0.453214	0.683766	1.000000	1.000000	1.000000	0.491228	1.000000

5-Conclusions and future suggestions

5-1-Conclusions

Evaluating the performance of different companies and industrial units is always an important issue to find the position of each decision-making unit in comparison with other units so that companies can find their weaknesses. In this paper, the network data envelopment analysis approach was used to achieve this goal. Since the number of input and output variables used in the model are high compared to the number of industrial units and this reduces the resolution of industrial units, the principal component analysis approach was used to reduce the data dimensions to an acceptable level. To implement the model, data from 26 stone cutting factories in Tehran, Isfahan and Qom were used. The main components were selected with the highest coverage variance and entered the model of network data envelopment analysis as inputs and outputs for different parts of the supply chain network of industrial units and the efficiency of each subnet for each industrial unit was calculated. Through the implementation of this model, the efficiency of each sector in industrial units is calculated and, if necessary, in addition to comparing industrial units with each other, different parts of these units can also be compared and found the weaknesses of each industrial unit in each network. In addition, the results obtained from this model, as shown in table 7, show an acceptable differentiation between industrial units for evaluating those units. However, if the main inputs and outputs were used as well as the raw data collected from the factories were used, the number of efficient units would increase and the model comparability between units would decrease. This result is shown in table 8.

Table 8. Efficiency of each subnet for stone industrial units – simple NDEA

DMUs	DMU1	DMU2	DMU3	DMU4	DMU5	DMU6	DMU7	DMU8	DMU9	DMU10	DMU11	DMU12	DMU13	DMU14	DMU15	DMU16	DMU17	DMU18	DMU19	DMU20	DMU21	DMU22	DMU23	DMU24	DMU25	DMU26
Purchase	1.000000	1.000000	0.176522	0.192135	0.278559	0.276936	0.318786	0.326759	0.097497	0.101197	0.114607	0.089381	0.24897	0.132339	0.351183	0.249824	0.071575	0.102865	0.258274	0.349109	0.080652	0.36902	1.000000	0.186719	0.199462	0.125023
Support	0.476755	1.000000	0.29827	0.460705	1.000000	1.000000	0.401688	0.642952	0.450823	1.000000	0.440955	0.427878	1.000000	0.576216	0.711361	0.455538	0.426595	0.54642	0.461843	0.397841	0.418554	0.604461	1.000000	1.000000	0.298088	0.423349
Producing	0.571542	0.181729	1.000000	0.581313	1.000000	0.383712	0.471282	0.332283	0.226737	1.000000	0.266789	0.314248	0.406789	0.385136	1.000000	0.688768	0.782405	1.000000	0.89003	0.889498	0.961581	0.927101	0.732959	0.947305	0.228893	0.677459
Sale	0.780803	0.513912	0.388397	0.863028	0.802007	0.258819	0.306507	0.551223	0.894393	1.000000	0.932001	1.000000	1.000000	0.977285	0.821856	0.864414	0.550895	0.593501	0.517827	0.453214	0.516536	0.683766	1.000000	1.000000	0.491228	1.000000

5-2-Future suggestions

Based on the results obtained from this article, some notices can be considered for future research. In the continuation of this research, other approaches on data envelopment analysis such as BCC or approaches based on auxiliary variables such as SBM can be used. This model has the ability to be implemented in different organizations and industries, so it can be used to calculate the efficiency of different units and compare them in an industry. Due to the implementation of the model in the stone industry and the innovations in this industry, in future research, more factories can be included in the model and industrial estates located in other cities can also be examined. The network considered in this study can also be expanded and the mines (as suppliers) can be added to the network.

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